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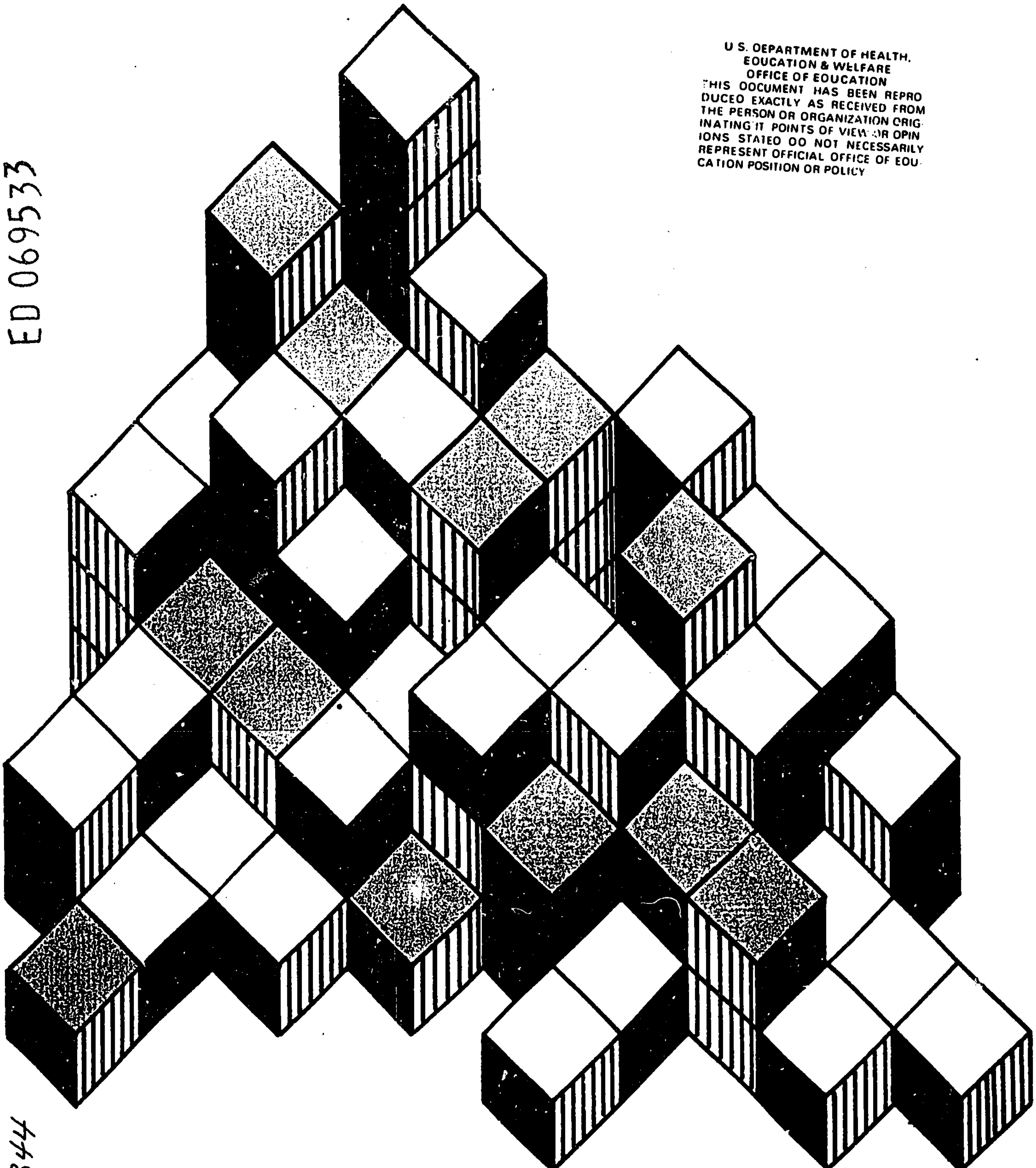
ABSTRACT

This geometric instructional unit concentrates on student use of three-dimensional manipulative aids. Rigorous definitions are avoided as students use categorical reasoning based on their own experiences. Through their own discovery of relationships, it is hoped students will become interested in geometry, aware of geometric forms in the world, and make better use of spatial perception. A teacher's guide is available. Related documents are SE 015 334 - SE 015 343 and SE 015 345 - SE 015 347. This work was prepared under an ESEA Title III contract. (LS)

GEOMETRIC EXCURSIONS



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GEOMETRIC EXCURSIONS

OAKLAND COUNTY MATHEMATICS PROJECT

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PREFACE

Fasten your seatbelts and prepare for takeoff! We are ready to begin an excursion into the world of geometry.

The first stop on our journey will be the city of Geopolis where we hope you will pay particular attention to the city's architecture. Before we leave Geopolis you and your classmates will be asked to construct a model of the city so that it may be referred to later on during the excursion.

As our tour takes you from lesson to lesson you will be given the opportunity to explore the properties of various space figures. This will help you identify various three-dimensional geometric shapes, distinguish one from another and discover relationships between them.

At one point we will stop to test some two-dimensional patterns to see if they can be formed into space figures. We hope this experience will help you develop your geometric imagination.

We will visit Sugar Lake City where cubes will present some puzzling problems. From cubes we'll move to the medium of clay and experiment with slicing through solids.

Before the end of the excursion, you will visit the realm of artists, architects and draftsmen where you will develop some techniques for picturing three-dimensional figures on a flat surface.

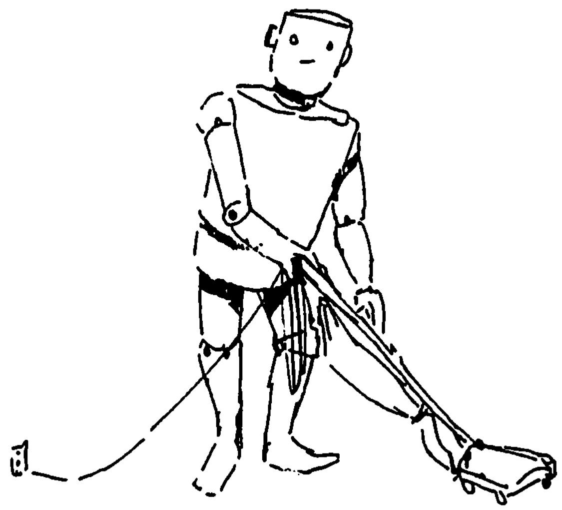
Our last stop will be the world of two dimensions where you will meet some of the shapes of Shadowland.

We hope you will enjoy this geometric excursion and we hope it will help you look at the world around you with a "geometric" eye.

TABLE OF CONTENTS

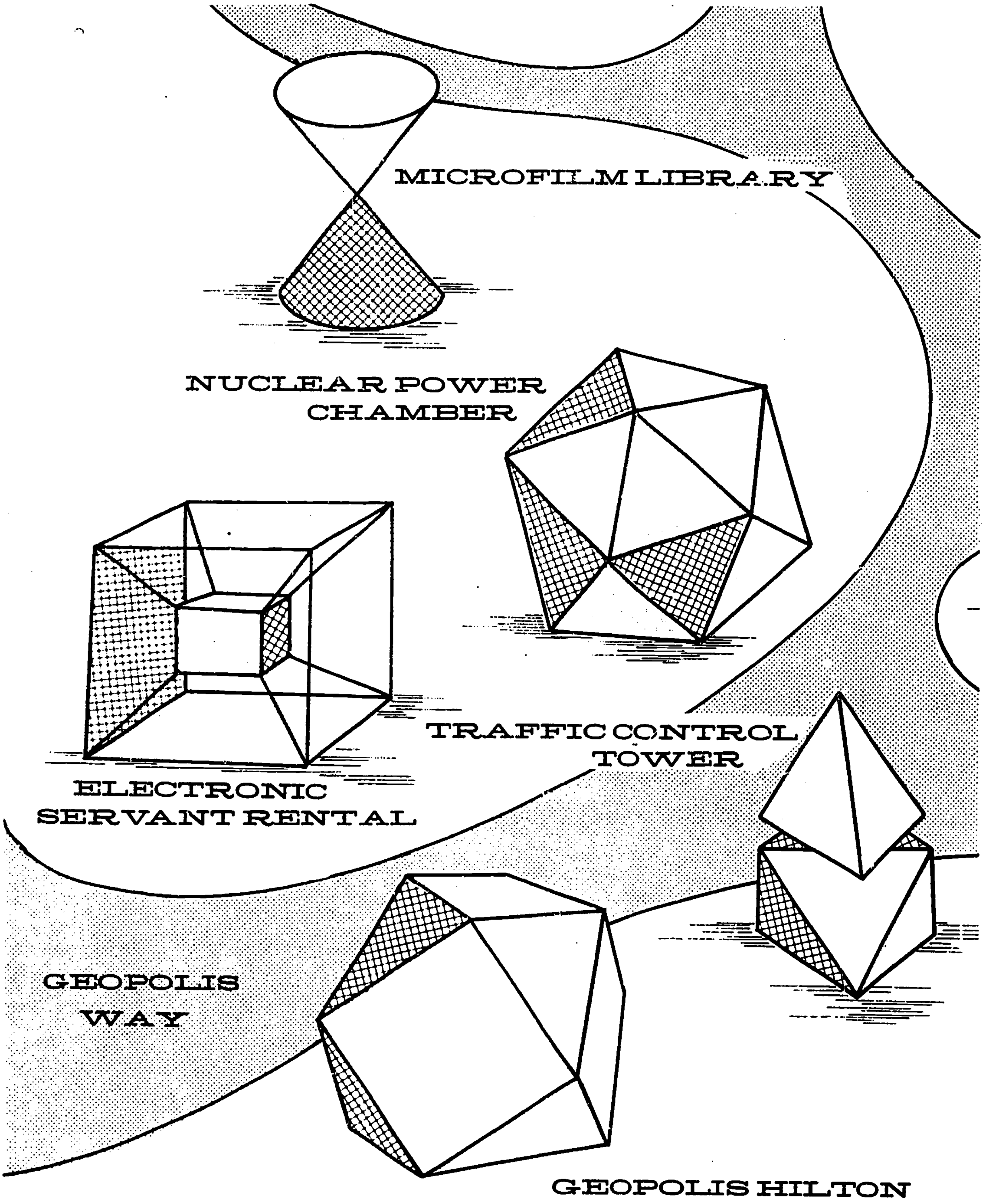
GEOPOLIS: A SPACE CITY OF 2001.

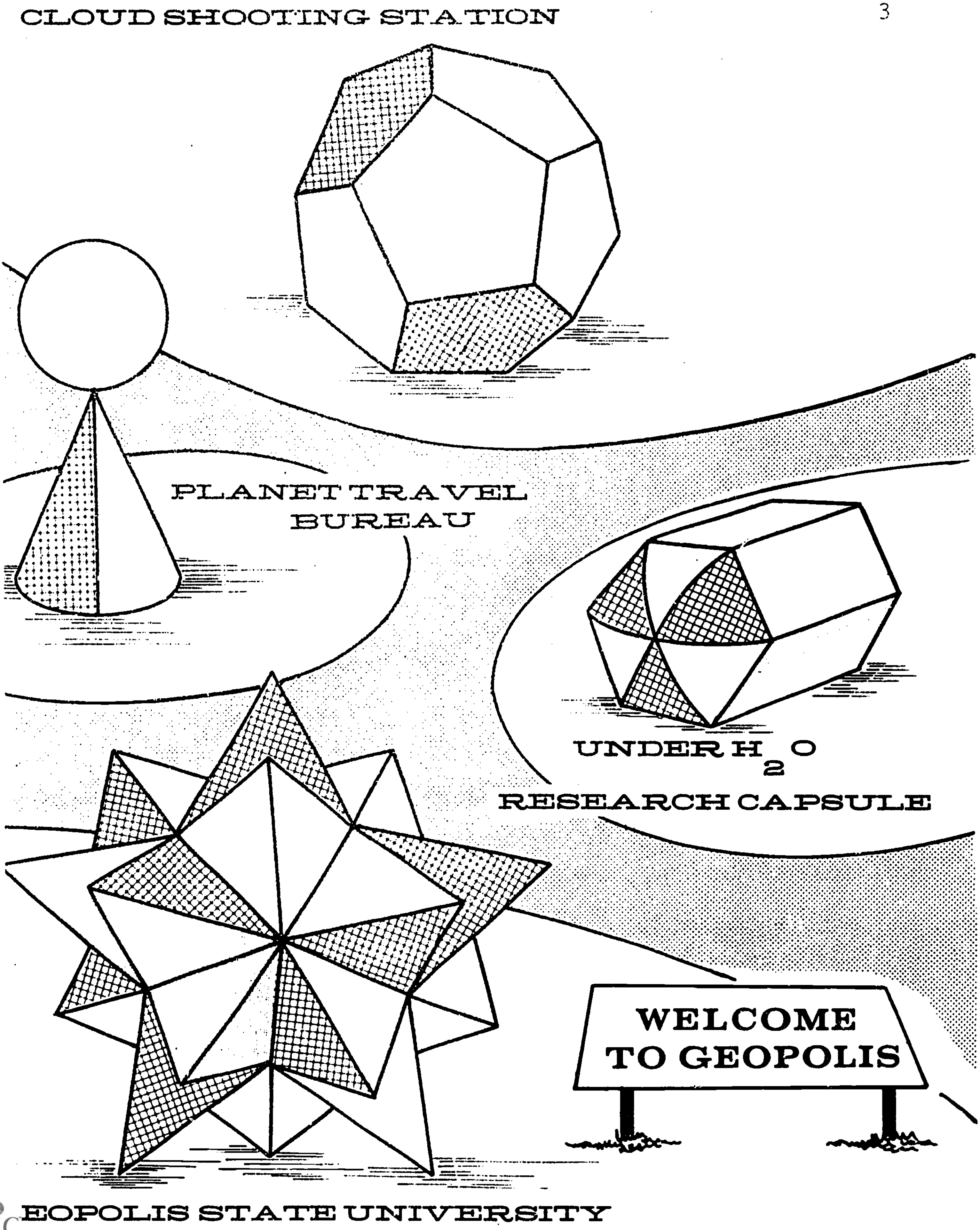
What will life be like in 2001 A.D.? Scientists and futurists predict that it will certainly be different from the world we now live in. They see man living a comfortable life while robots act as servants. They even predict that man will take a pill to improve his "brain-power". Wouldn't that be great – intelligence pills?



People will travel to and from work on monorails or they will drive their cars onto electric highways which resemble huge conveyor belts and glide to work. Hopefully, this will eliminate the agonizing traffic jams we now face on our so-called "expressways". For those people who dislike city-life, they will have an opportunity to get away from it all. They can vacation in space or under the sea. They can even live and work in under-water dwellings which will resemble huge plastic bubbles.

Those people who prefer not to give up star-gazing for bubble-watching will live in huge cities. The city centers will include offices, schools, research centers and control towers. Houses and apartments will be built outside the city center areas. Looking ahead to the year 2001, we tried to visualize one of these city center areas. We created one which we call Geopolis. Study the layout on pages 2 and 3. Then answer the questions on top of page 4.


MICROFILM LIBRARY
NUCLEAR POWER CHAMBER
ELECTRONIC SERVANT RENTAL
TRAFFIC CONTROL TOWER
GEOPOLIS WAY
GEOPOLIS HILTON


CLOUD SHOOTING STATION
PLANET TRAVEL BUREAU
UNDER H_2O RESEARCH CAPSULE
WELCOME TO GEOPOLIS
EOPOLIS STATE UNIVERSITY

THINK ABOUT IT ...

1. If you were in charge of designing Geopolis, what additions or deletions would you make?

2. Where (if ever) have you seen the geometric shapes on pages 2 and 3 used in present architecture? Are they commonly used? Why or why not?

3. What other unusual geometric shapes have you seen used in present day architecture?

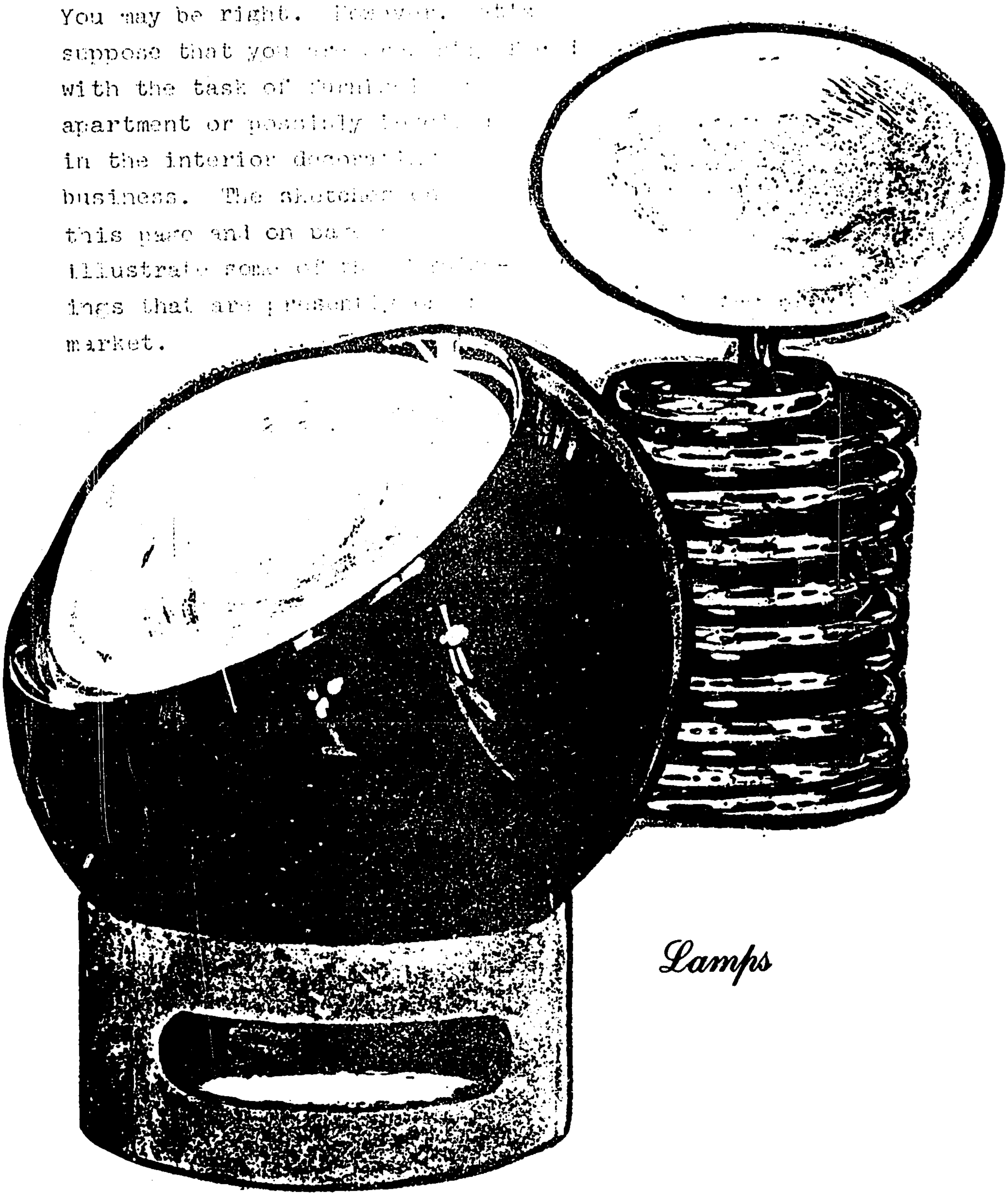
4. What geometric shapes are most commonly used in designing the buildings of today? Why do you think they are used so often?

CLASS ACTIVITY

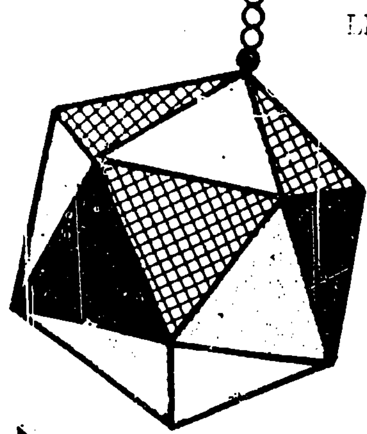
The geometric shapes used in designing Geopolis have been purposely selected so that we may study them in future lessons on this booklet. You will work in teams to build a model of Geopolis. Your teacher will pass out the job assignment cards and materials.

SHAPES OF THE 70'S

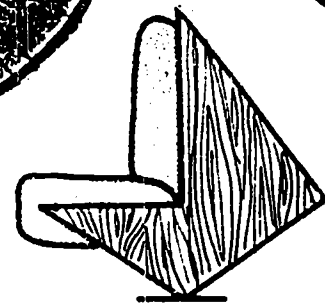
You probably felt that the architectural design of Geopolis was far-fetched and highly [illegible] even for 2001 A.D.. You may be right. However, [illegible] suppose that you [illegible] with the task of [illegible] apartment or possibly [illegible] in the interior decora[illegible] business. The sketches on this page and on [illegible] illustrate some of [illegible] ings that are presently [illegible] market.



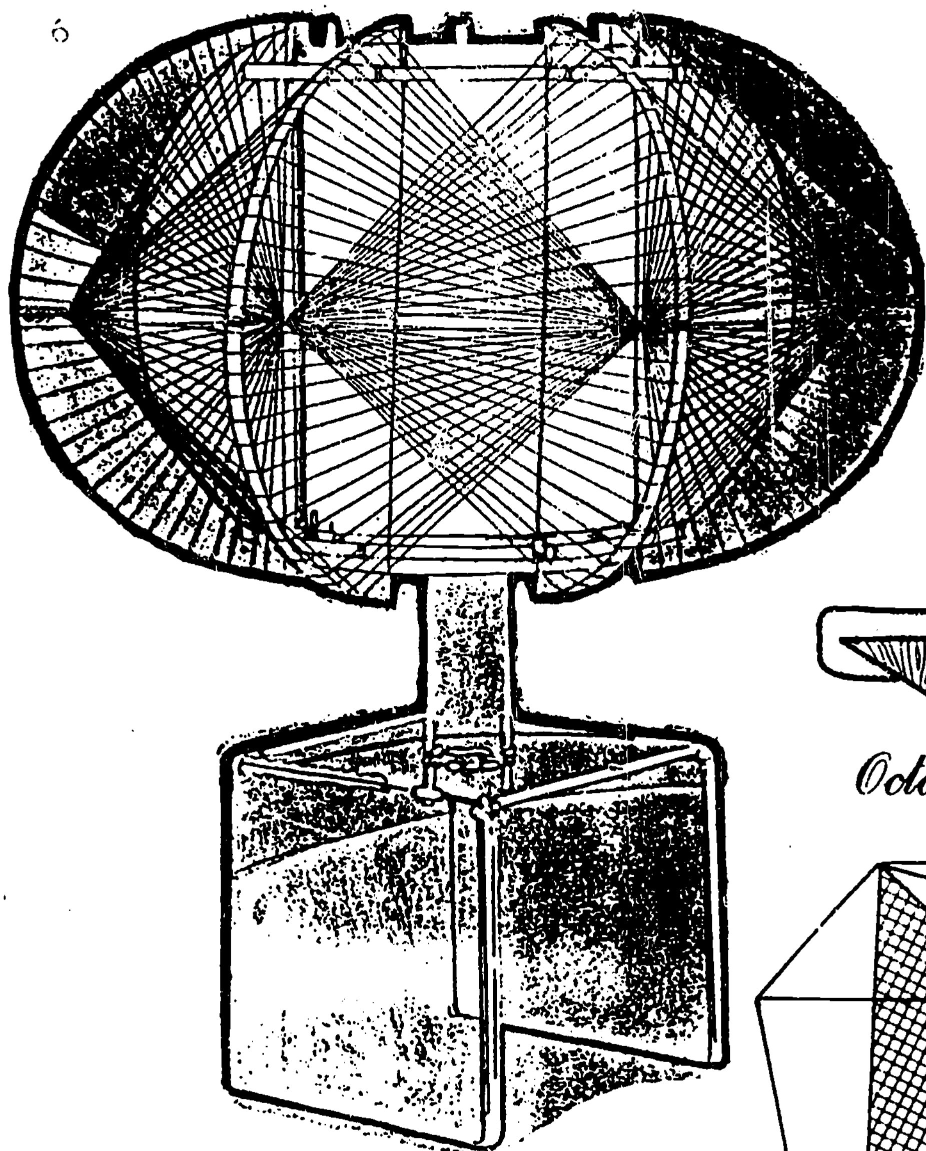
Lamps



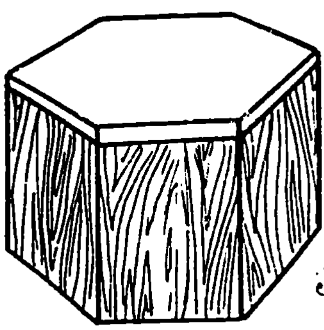
Icosahedron Lamp



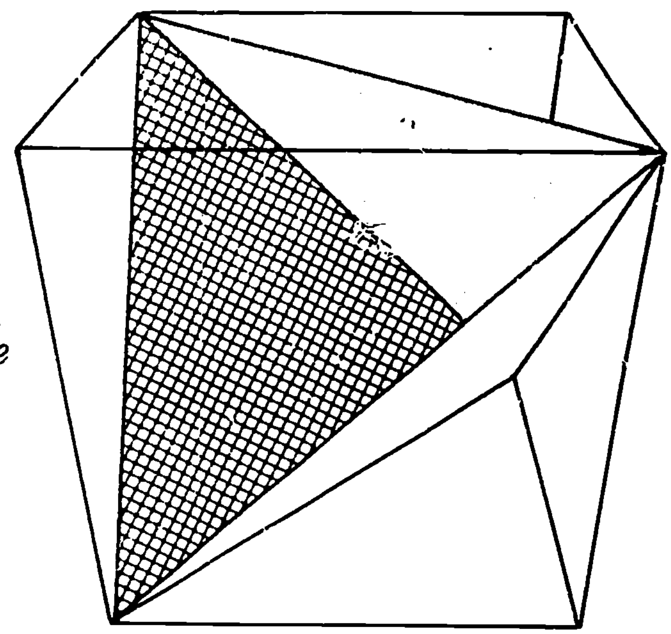
Octahedron Chair



Lamp & Table



Hexagon Table

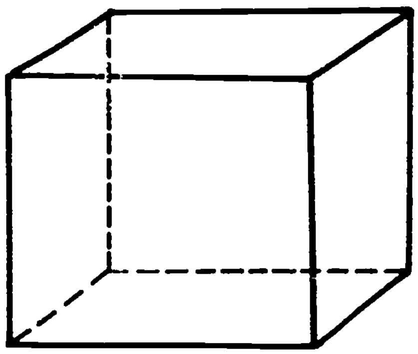


Cube-Tetrahedron Table

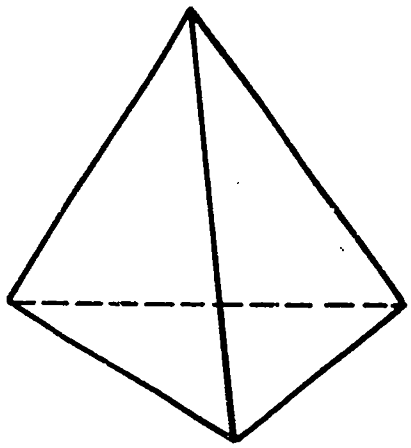
Do these furnishings bear any resemblance to the geometric shapes on pages [illegible] and [illegible]? If you don't believe the sketches, visit a couple of the furniture stores in your area.

SORTING OUT SOLIDS

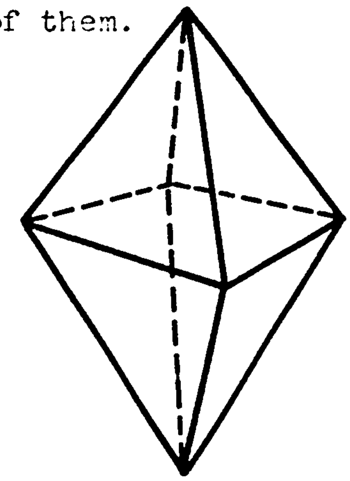
Although the shapes used in designing the buildings of Geopolis look very modern and "way-out", they are patterned after shapes which have been known to man for many years. It is said that the ancient Egyptians knew four of them.



HEXAHEDRON

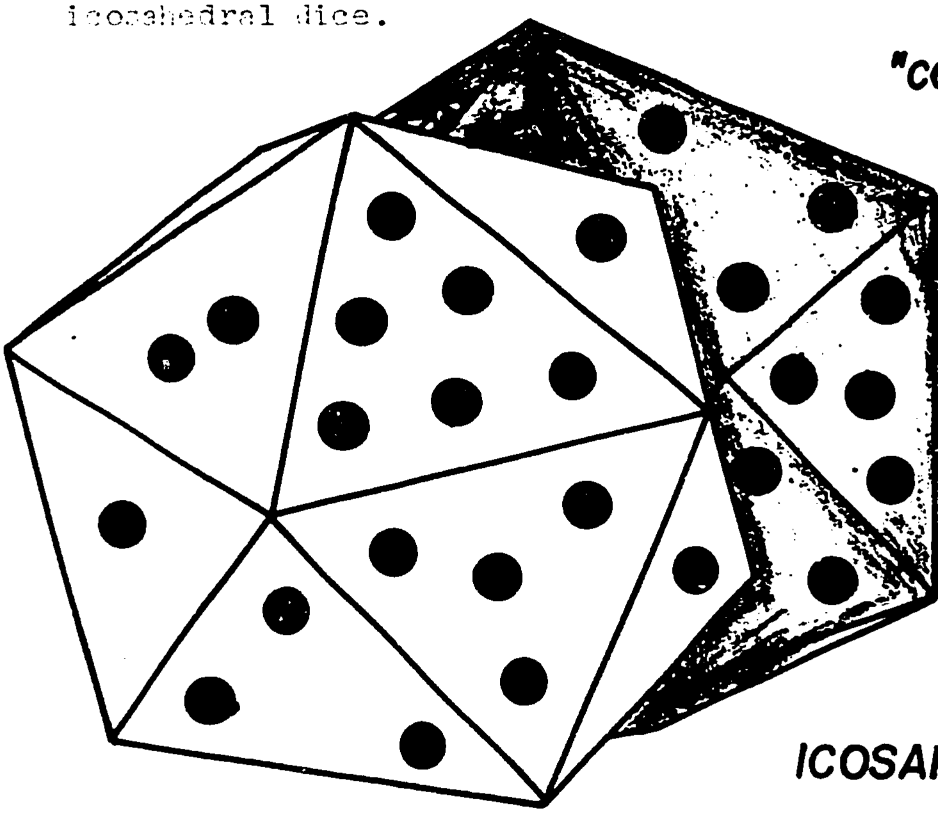


TETRAHEDRON



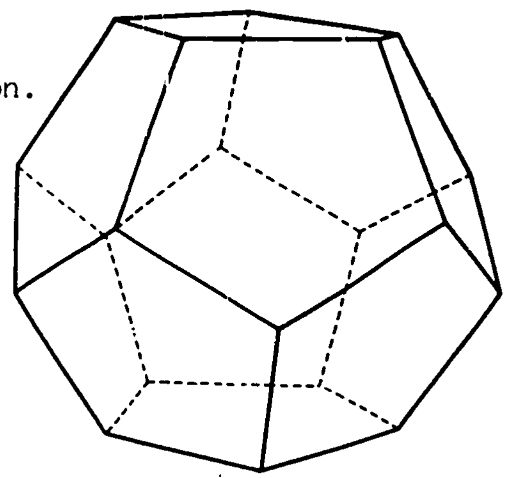
OCTAHEDRON

The British Museum has on display a pair of Egyptian icosahedral dice.



ICOSAHEDRONS

The Pythagoreans, a secret society of the Greeks (540 B.C.) discovered the regular dodecahedron.

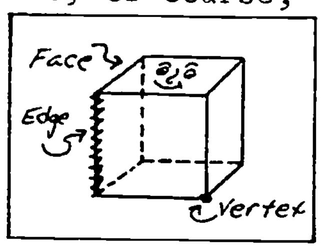


Why do you suppose these shapes were given such strange sounding names? (Use a dictionary to find out.) Which of these five shapes is more commonly known as a cube?

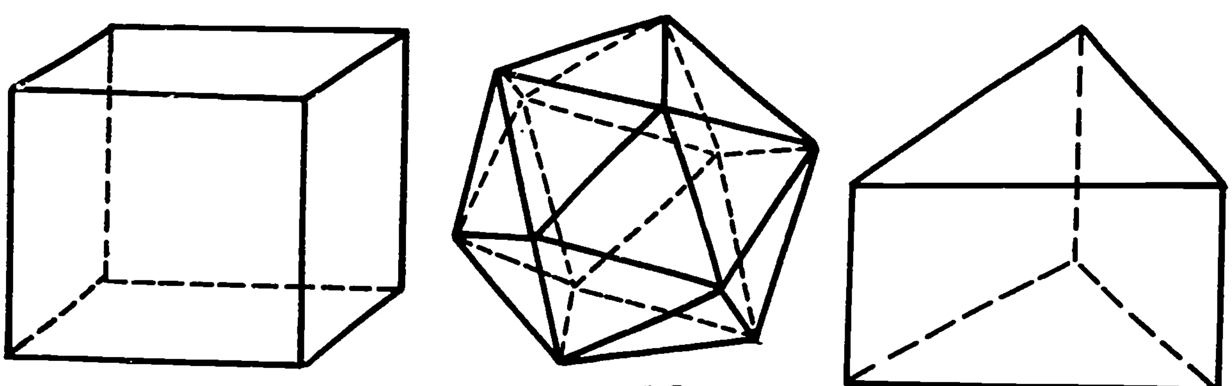
These five figures (**TETRAHEDRON, CUBE, OCTAHEDRON, ICOSAHEDRON, DODECAHEDRON**) belong to a set of three-dimensional geometric figures called Polyhedra.

What are polyhedra? Use the next three sets of figures to answer this question on your own. (Note: the dotted lines show sides which would be hidden from view unless, of course, the figure was made out of straws.)

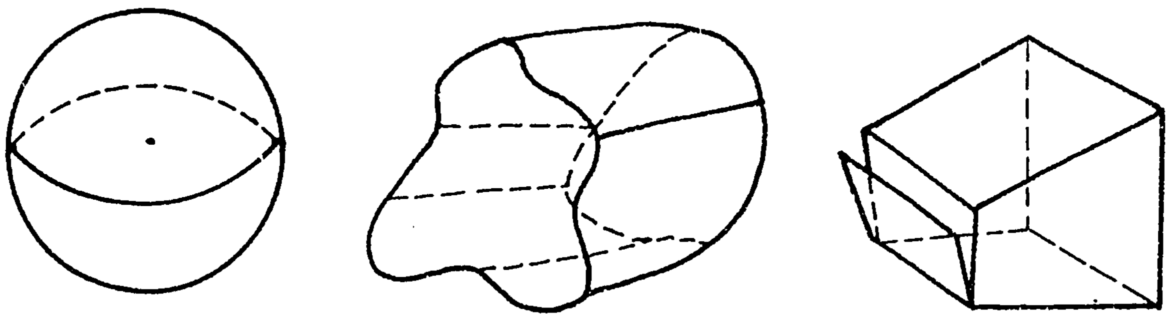
KEEP THESE TERMS IN MIND



THESE ARE POLYHEDRA.

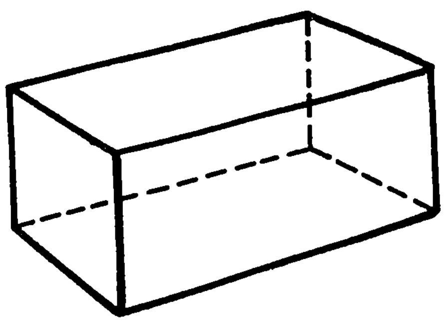


THESE ARE NOT POLYHEDRA.

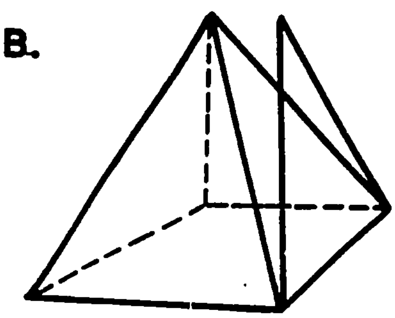


WHICH OF THE FOLLOWING ARE POLYHEDRA?

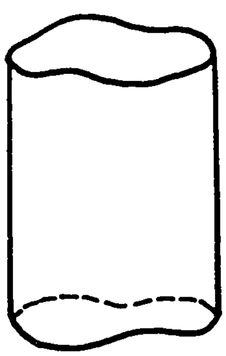
A.



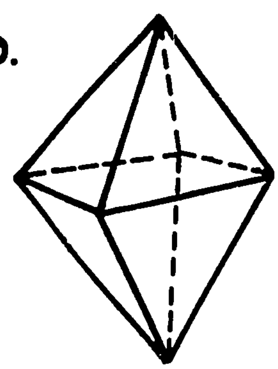
B.



C.



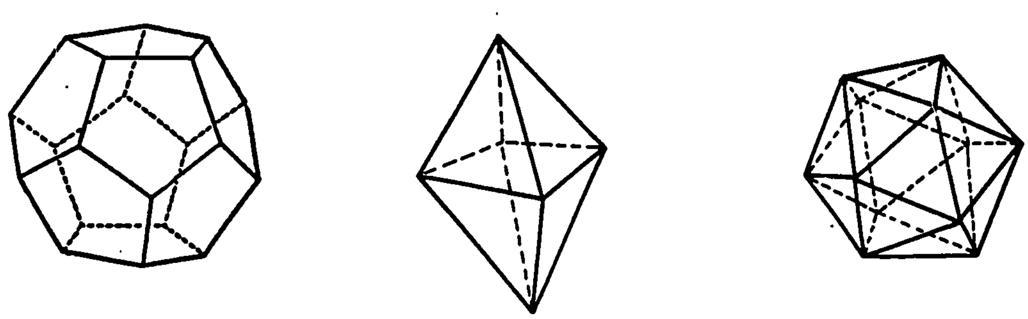
D.



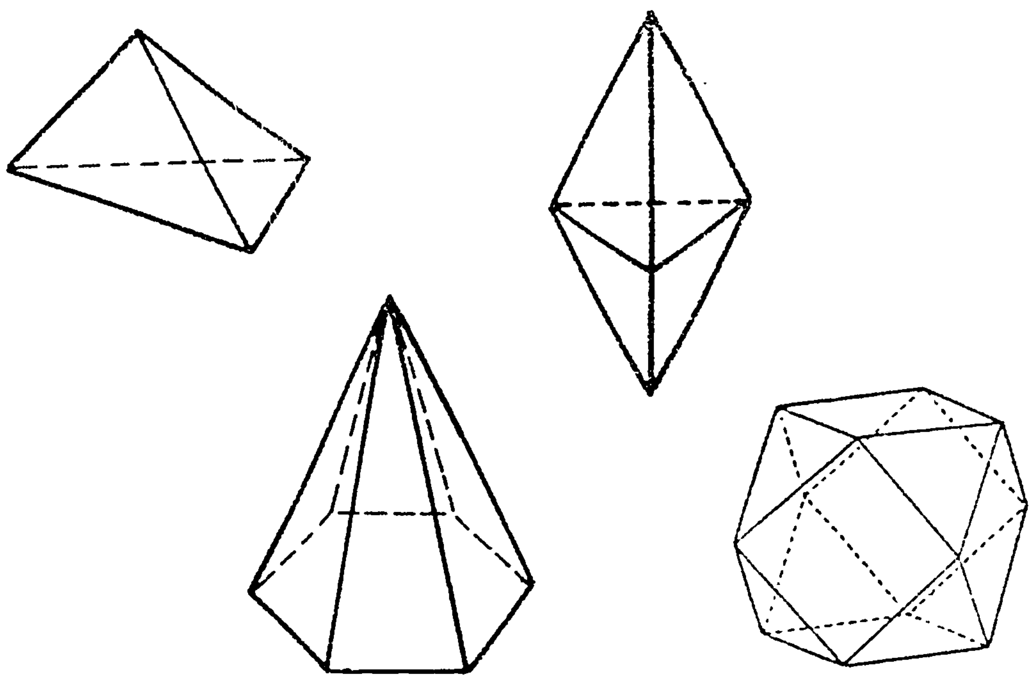
Jot down some of the properties you would use to identify a polyhedron and go on to the next page.

The set of polyhedra includes a special set of space figures called regular polyhedra. See if you can figure out what the properties of a regular polyhedron are.

THESE ARE REGULAR POLYHEDRA.

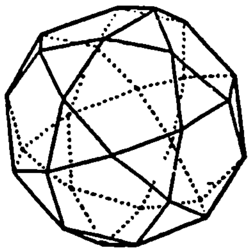


THESE ARE NOT REGULAR POLYHEDRA.

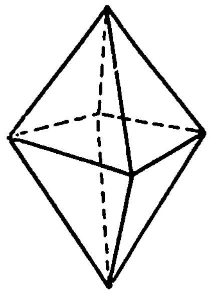


WHICH OF THE FOLLOWING ARE REGULAR POLYHEDRA?

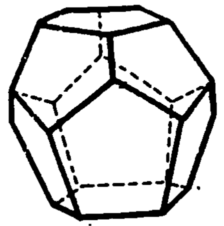
a.



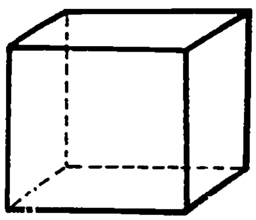
b.



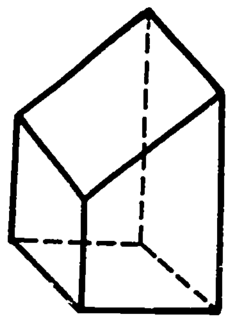
c.



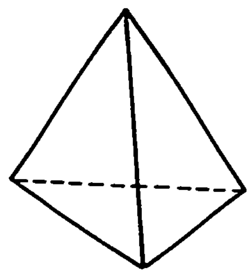
d.



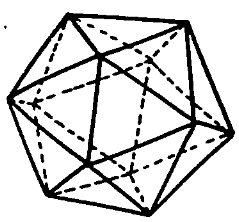
e.



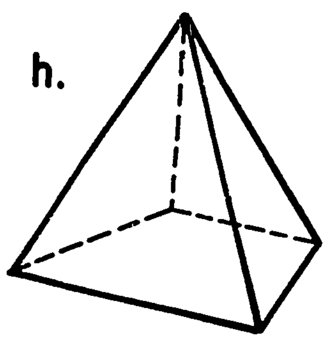
f.



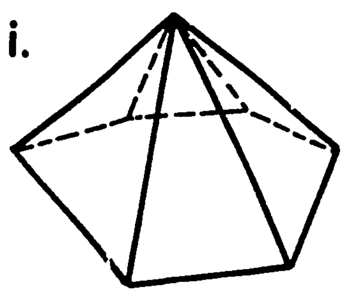
g.



h.



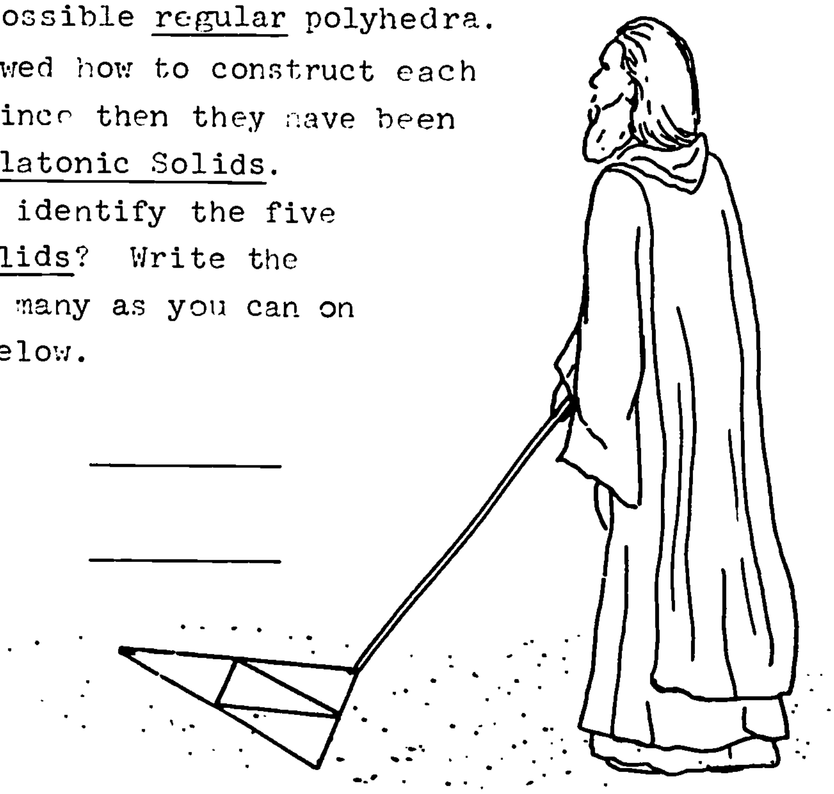
i.



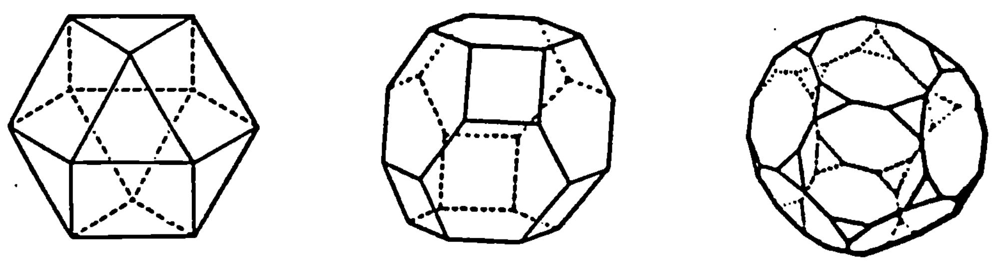
Jot down the ideas you used in making your decision and proceed to page 12.

Plato (400 B.C.), a Greek philosopher, said that there are only five possible regular polyhedra. He also showed how to construct each of them. Since then they have been named the Platonic Solids.

Can you identify the five Platonic Solids? Write the names of as many as you can on the lines below.

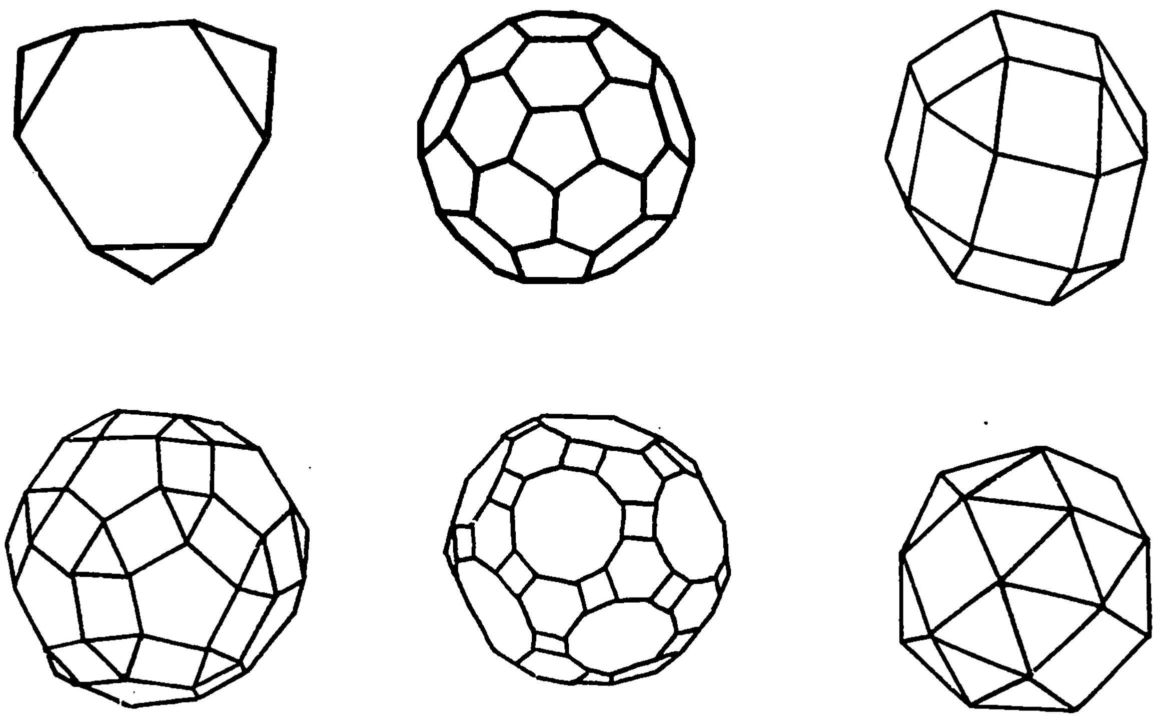


Later on, Archimedes (278 B.C.) added thirteen more figures to the set of polyhedra. They are included in the set of semi-regular polyhedra. Three of the thirteen are pictured below:

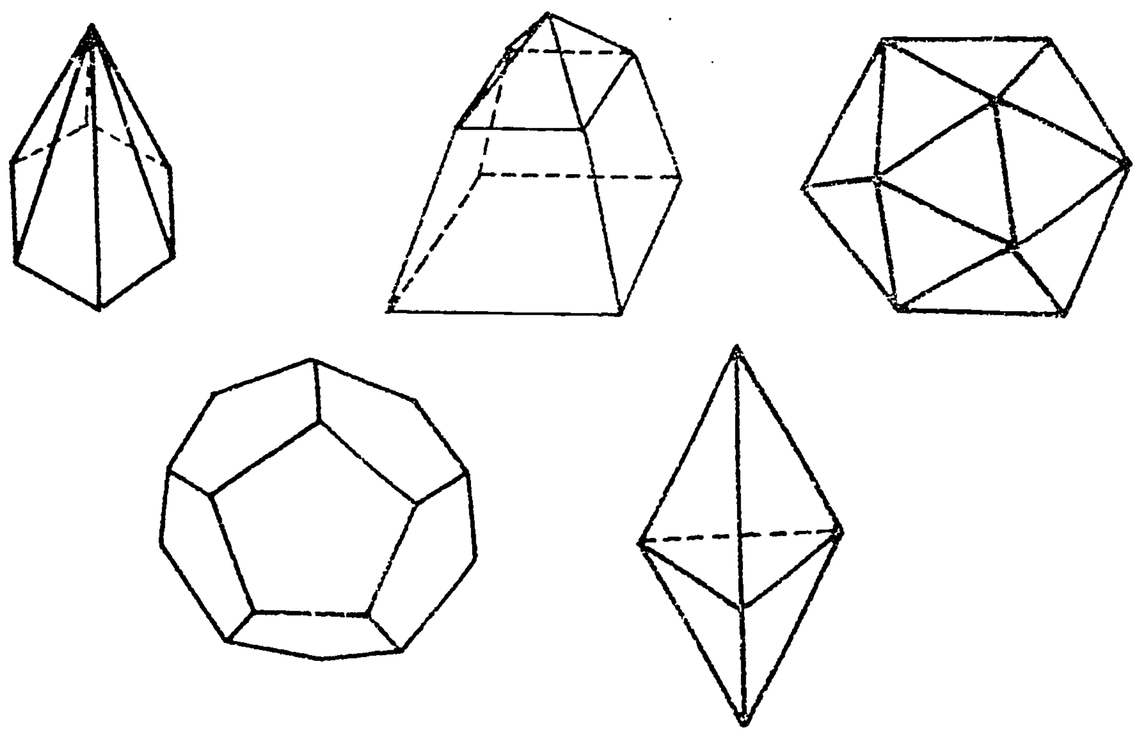


Can you describe a semi-regular polyhedron? If not, the next two pages will help you. If you think you can, record your ideas and then check them by studying the figures on the next two pages.

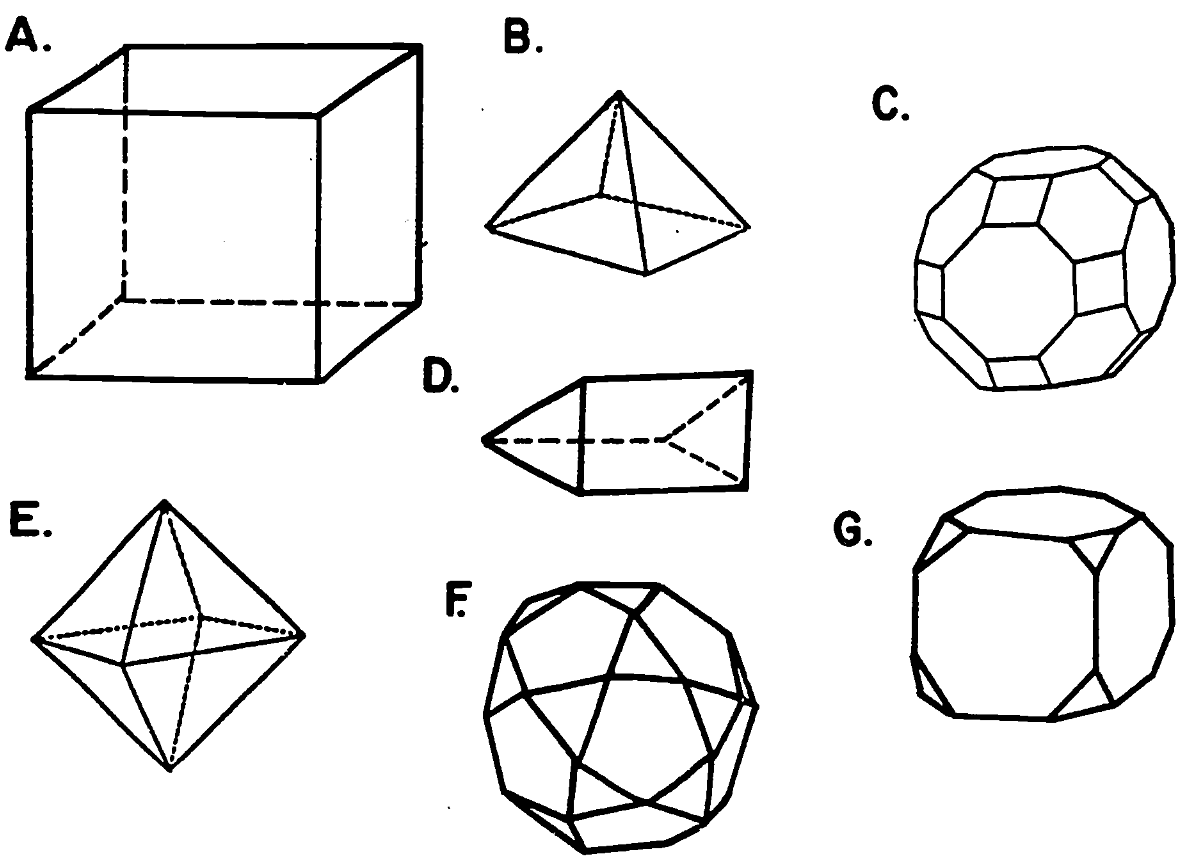
THESE ARE <u>SEMI-REGULAR POLYHEDRA</u>.



THESE ARE <u>NOT SEMI-REGULAR POLYHEDRA</u>.



WHICH OF THE FOLLOWING ARE SEMI-REGULAR POLYHEDRA?



Checklist

Check those properties which you think are characteristic of all semi-regular polyhedra.

_____ 1. The same number of edges meet at each vertex.

_____ 2. Made up of at least two different-shaped faces.

_____ 3. Faces are polygons.

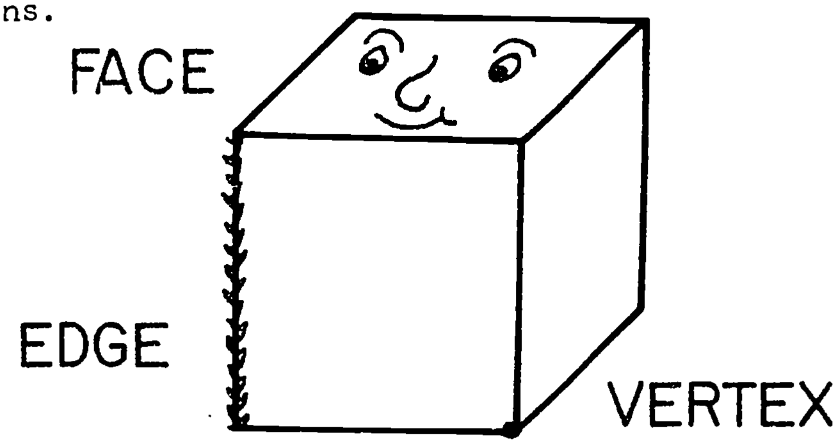
_____ 4. Faces are regular polygons.

_____ 5. Like faces are the same size.

_____ 6. Like faces are not necessarily the same size.

THINK ABOUT IT ...

Use the terms face, edge and vertex in answering the following questions.

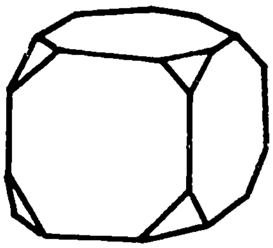


1. If a three-dimensional figure is a polyhedron what must be true of its faces? of its edges?
2. What are some of the other characteristics of a polyhedron?
3. What is the least number of vertices a polyhedron can have? the least number of faces?
4. What is true about the edges of a regular polyhedron? about the faces?
5. What makes a semi-regular polyhedron different from a regular polyhedron? How are they the same?
6. What do you suppose are the characteristics of an irregular polyhedron?
7. What is the greatest number of faces a polyhedron can have?

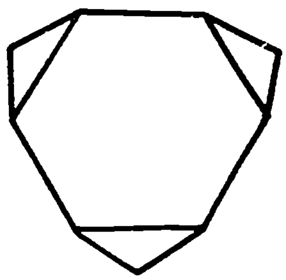
There are many other polyhedra. Some look like stars. Others are called pyramids and prisms. All the polyhedra play an important part in shaping the three-dimensional world we live in.

ON YOUR OWN

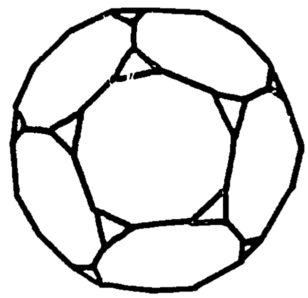
1. Some of the Archimedean (semi-regular) solids are formed by slicing off the corners of the five Platonic Solids. Match each of the semi-regular figures below with the Platonic Solid (on the next page) which was used as its basis. If you're not really sure — <u>guess</u>. (HINT — The numbers corresponding to the figures on page 17 can be used more than once.)



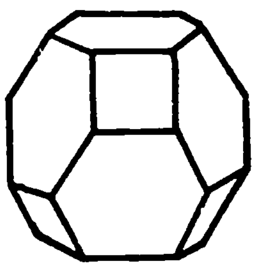
a. ______



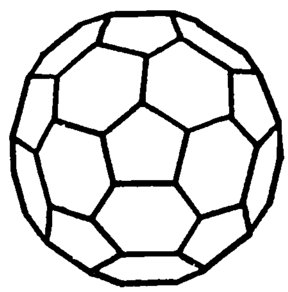
b. ______



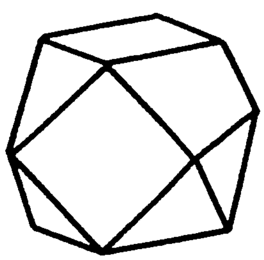
c. ______



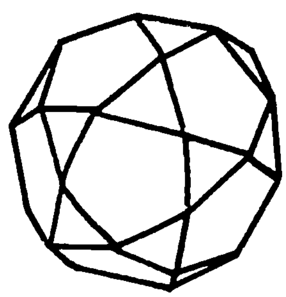
d. ______



e. ______



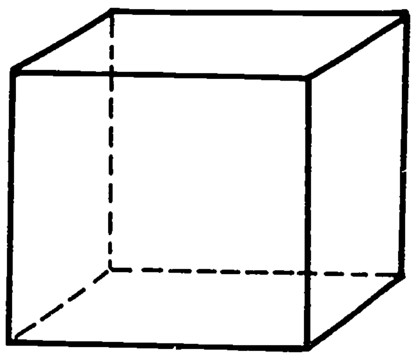
f. ______



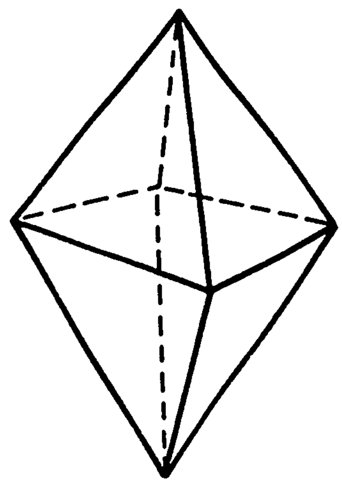
g. ______

PLATONIC SOLIDS

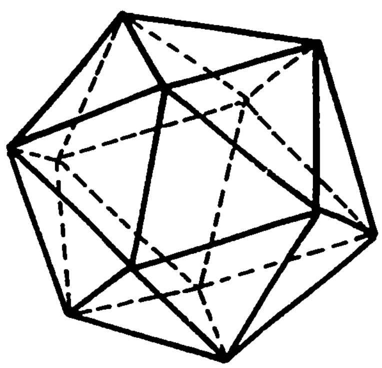
1. CUBE



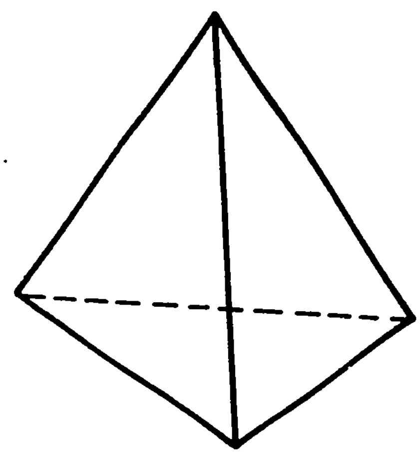
2. OCTAHEDRON



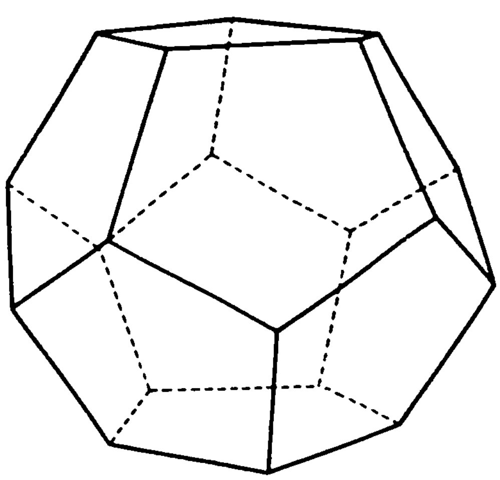
3. ICOSAHEDRON



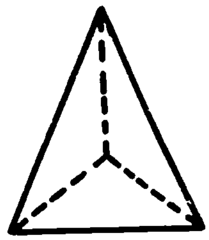
4. TETRAHEDRON



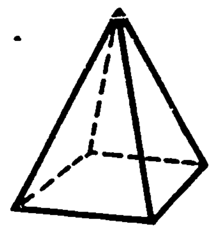
5. DODECAHEDRON



2. The following are pyramids with which you are probably familiar:

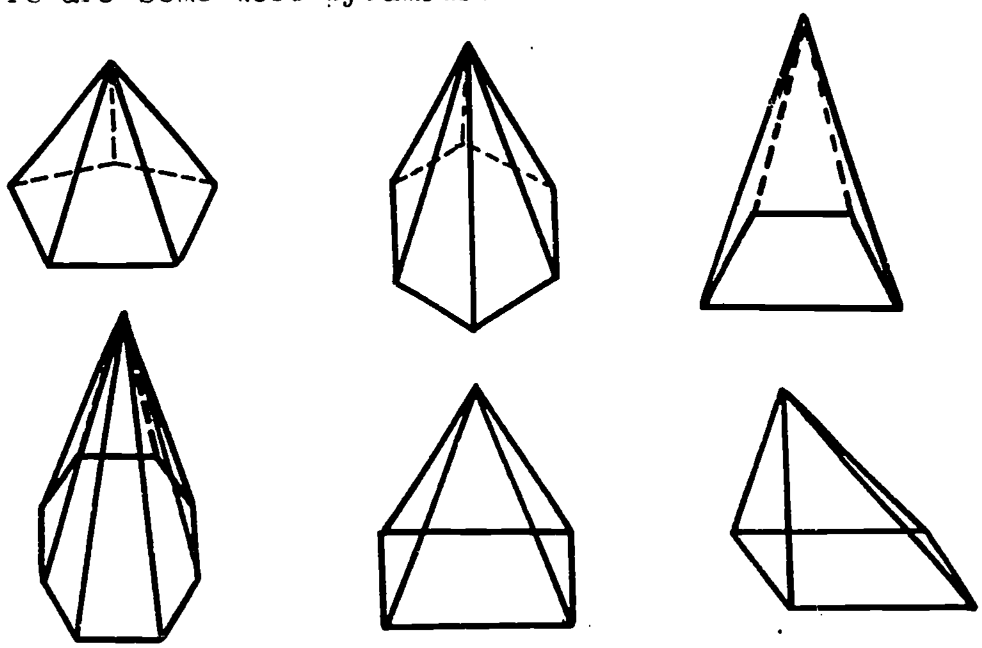


Triangular Pyramid



Square Pyramid

Here are some more pyramids:



a. Which of the following are characteristics of a pyramid? __________

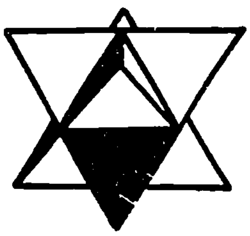
   (1) The base is a polygon.
   (2) All the edges meet at a point.
   (3) All faces other than the base are triangles.
   (4) The base is a regular polygon.
   (5) All but one vertex lie on the base.

b. (1) What is the smallest number of edges that the base of a pyramid can have? __________

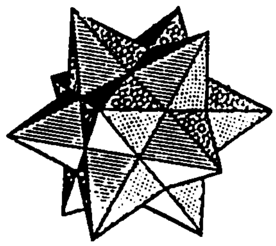
   (2) What is the largest number? __________

c. How would you distinguish one pyramid from another?

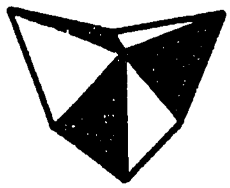
3. The polyhedra pictured below are called star polyhedra. These five were formed by building a pyramid on each face of one of the <u>Platonic Solids</u> (see page 17). Identify the Platonic Solid which was used to form each one. Use the letters T, C, O, I, D to identify the one which you think was used. If you're not sure — Guess! (Hint — Do the number of edges on each face of Plato's polyhedra give you any clues?)



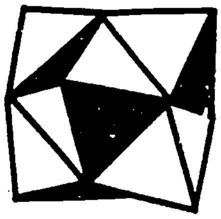
a.__________



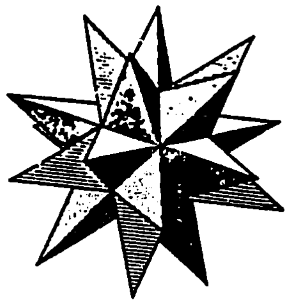
b.__________



c.__________



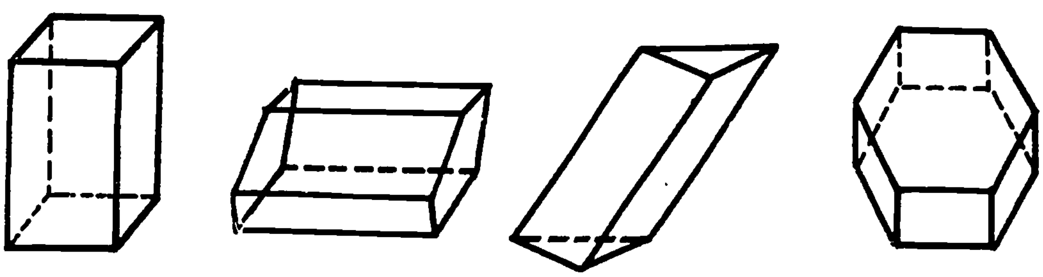
d.__________



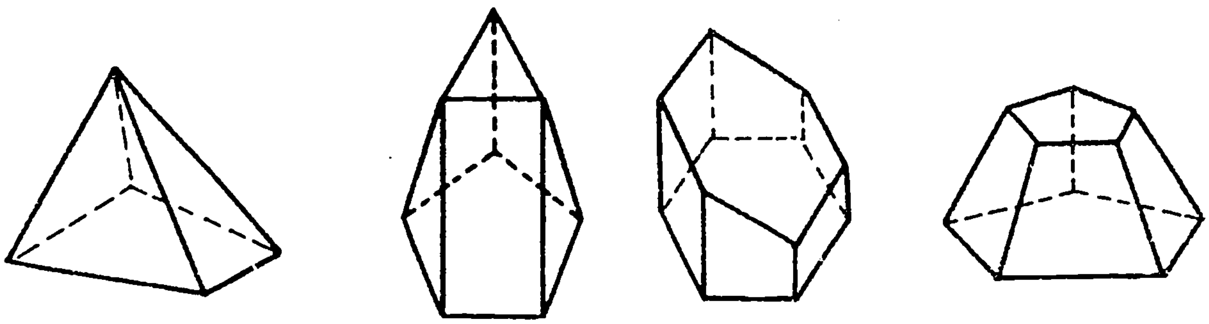
e.__________

4. Now let's look at a subset of polyhedra which are known as prisms.

Each of the following are prisms:



These are not prisms:



a. Use the preceding sets of figures to decide which of the following statements are true and which are false.

_____ (1) All the faces of a prism are the same shape and size.

_____ (2) At least two faces of a prism are parallel and also the same shape and size.

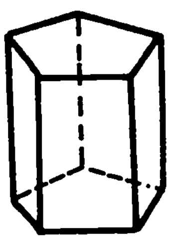
_____ (3) All but two faces of a prism are rectangles.

_____ (4) A pyramid can also be a prism.

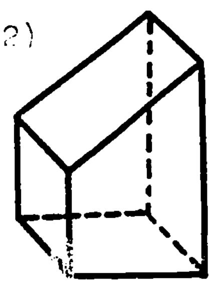
_____ (5) All but two faces of a prism are parallelograms.

b. Which of the following are prisms?

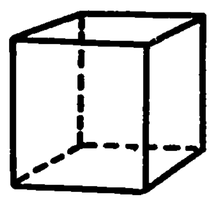
(1)



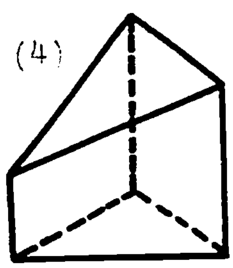
(2)



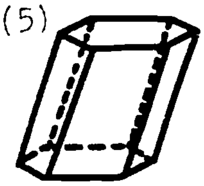
(3)



(4)



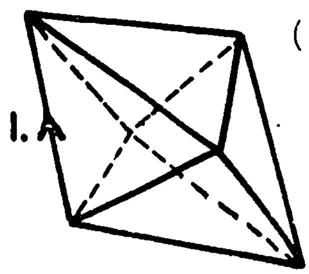
(5)



c. Describe a prism in your own words.

5a. There are gas fumes escaping from the Traffic Control Tower. The leak is somewhere along one of the edges in the bottom structure. A trouble shooter is sent out to find the crack in the edge. He will move along every edge on the outside of the building. Is it possible for him to move along every edge without going over the same edge twice? If so, show how he can do it. Use arrows and numbers to show a path he might travel.

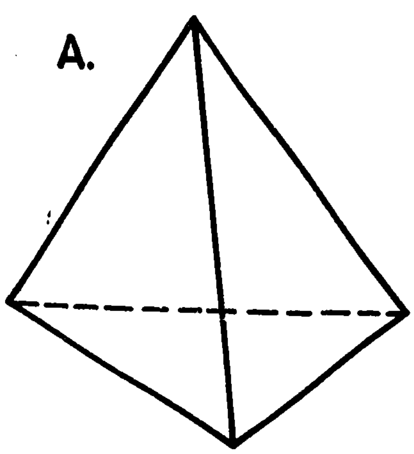
(Caution – use a pencil.)



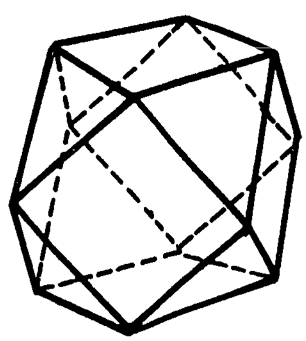
b. Suppose each of the figures below represents a building. For which of the figures is it possible to move along every edge without going over the same edge twice?

______________________ (If you think the task can be accomplished, show a path which can be taken.)

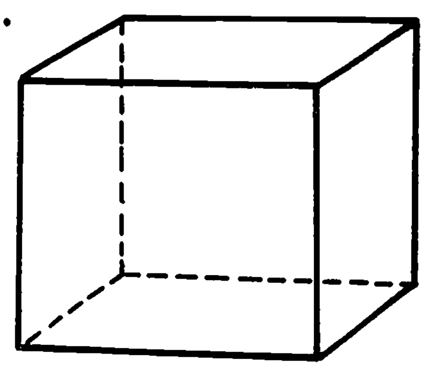
A.



B.



C.



How can you tell by looking at a figure whether it is possible to move along every edge without going over any edge twice?

✓ Point

1. A polyhedron is: ____________

   (select the best answer)

   a. a simple, closed space figure made up of polygons.

   b. any space figure.

   c. any space figure made up of polygons.

2. A regular polyhedron has: ____________

   (select the best answer)

   a. faces which are the same size and shape.

   b. the same number of edges meeting at each vertex.

   c. edges which are the same length.

   d. both a and b. e. both b and c.

3. Mark the following true (T) or false (F):

___ a. There are exactly four regular polyhedra.

___ b. The edges of a semi-regular polyhedron are all the same length.

___ c. One of the star polyhedra can be formed by building a triangle on each face of a cube.

___ d. The same number of edges meet at each vertex of a semi-regular polyhedron.

___ e. All but two faces of a prism are rectangles.

___ f. The base of a pyramid must have at least four edges.

___ g. All but one vertex of a pyramid lie on the base.

TAKING A NUMBER BREAK

In the following problems, some of the digits in the numbers have been replaced by letters. Figure out which of the digits 0-9 each letter represents. Each letter stands for one and only one digit and different letters stand for different digits.

EXAMPLES:

I. $\begin{array}{r} ZC \\ +\ \underline{17} \\ 35 \end{array}$ Z = 1, C = 8 $\begin{array}{r} 18 \\ +\ \underline{17} \\ 35 \end{array}$

II. $\begin{array}{r} MM \\ -\ \underline{8} \\ 2J \end{array}$ M = 3, J = 5 $\begin{array}{r} 33 \\ -\ \underline{8} \\ 25 \end{array}$

a. $\begin{array}{r} 53 \\ -\ \underline{NE} \\ 17 \end{array}$ N = ___ E = ___

b. $\begin{array}{r} 34 \\ +\ \underline{U} \\ A1 \end{array}$ U = ___ A = ___

c. $\begin{array}{r} WL \\ -\ \underline{W} \\ S \end{array}$ W = ___ L = ___ S = ___

d. $\begin{array}{r} SS \\ +\ \underline{W} \\ WLL \end{array}$ S = ___ W = ___ L = ___

e. $\begin{array}{r} UTT \\ +\ \underline{D2} \\ 774 \end{array}$ U = ___ T = ___ D = ___

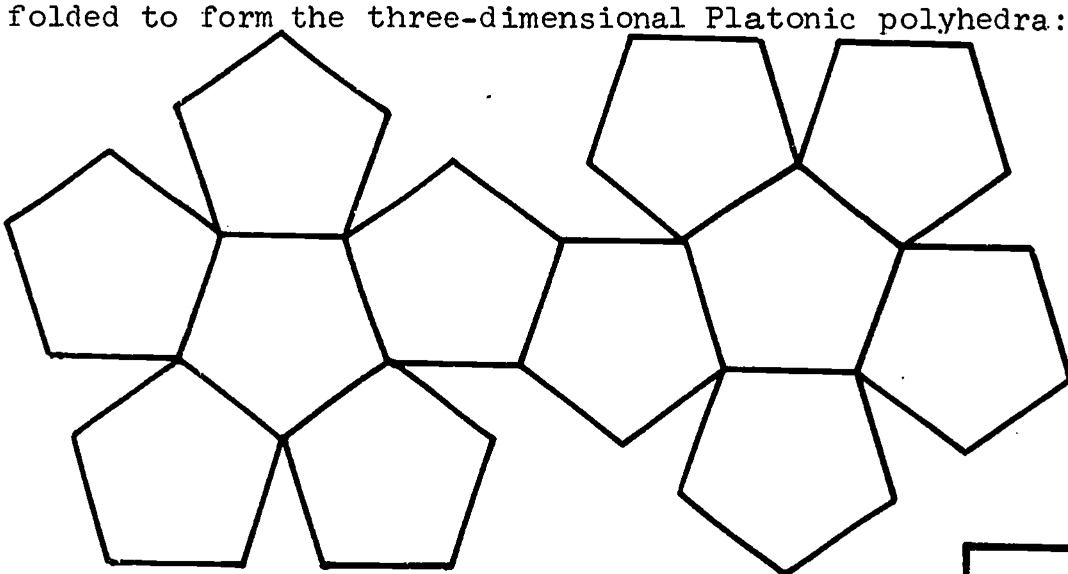
f. $\begin{array}{r} IW3 \\ -\ \underline{3IW} \\ A32 \end{array}$ I = ___ A = ___ W = ___

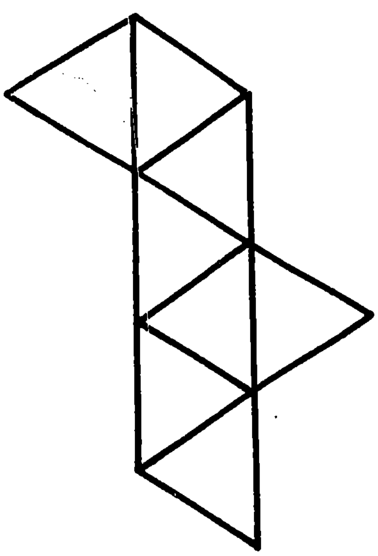
Figure out the following message by replacing the numerals with letters. Use your answers to the problems above to make the substitutions. (Caution - Ø stands for the letter Ø and 0 stands for zero.)

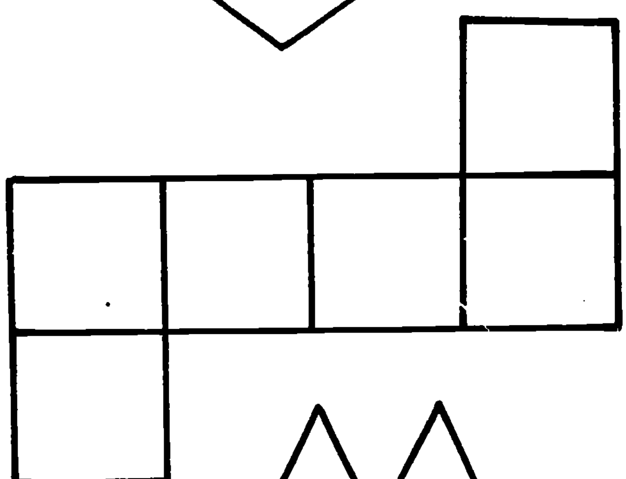
Y Ø 7   4 R 6   2 H 6   1 8 3 3 6 R
Ø F   4 3   4 0 0   6 X P 6 3 9 6   P 4 8 5
1 6 6 K 6 3 5   8 3   B 6 4 7 2 8 F 7 0
5 Ø 1 3 2 Ø 1 3   G 6 Ø P Ø 0 8 9

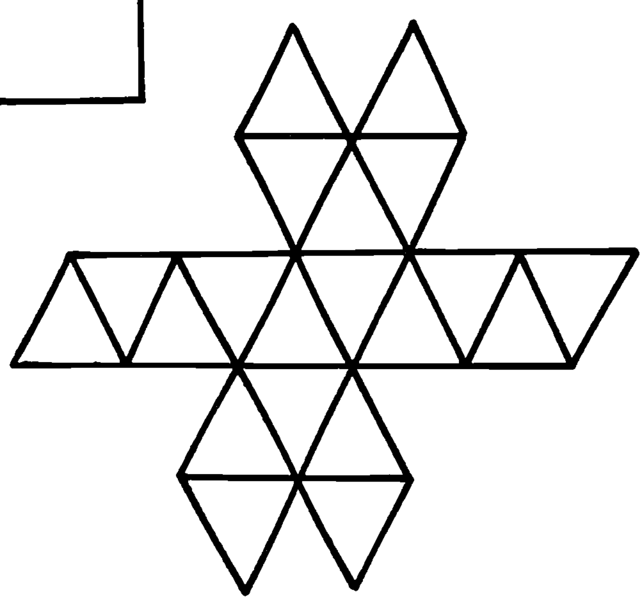
POLY PATTERNS

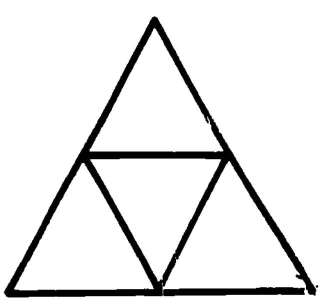
The following are two-dimensional patterns which can be folded to form the three-dimensional Platonic polyhedra:





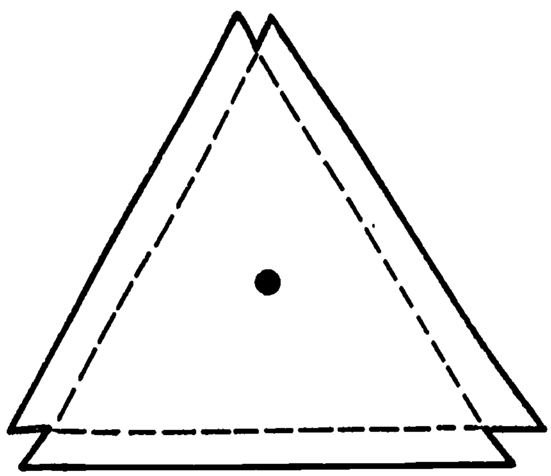






CLASS ACTIVITY

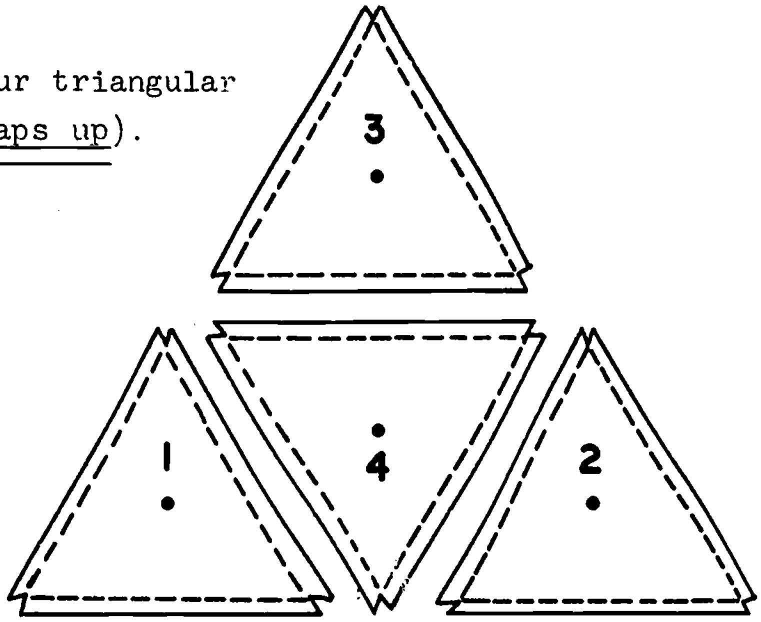
1. Cut out the triangular, square and pentagonal panels on the following inserts (A, B, C, C). Fold along the dotted lines to form flaps.



2. Pool your panels with those of two of your classmates. Arrange the panels (flaps up) to form patterns exactly like those on page 24. Before your patterns can be folded to form models of the Platonic Solids, the panels which touch will have to be connected. You will need a supply of rubber bands. Start with the pattern which is like the one in the lower left-hand corner of page 24. One person in the group should hold the pattern in place while the other two put the rubber bands on. After this has been completed, the pattern can be folded to form a tetrahedron. If the above instructions are not clear enough, study the step-by-step instructions which are on the next page.

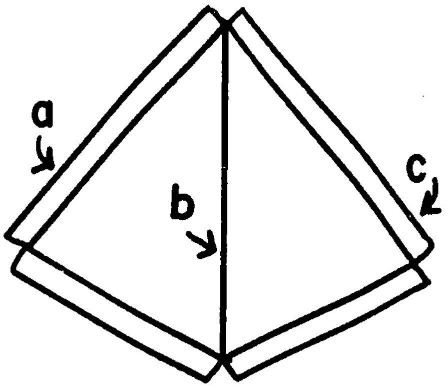
EXAMPLE:

a. Arrange four triangular panels (flaps up).

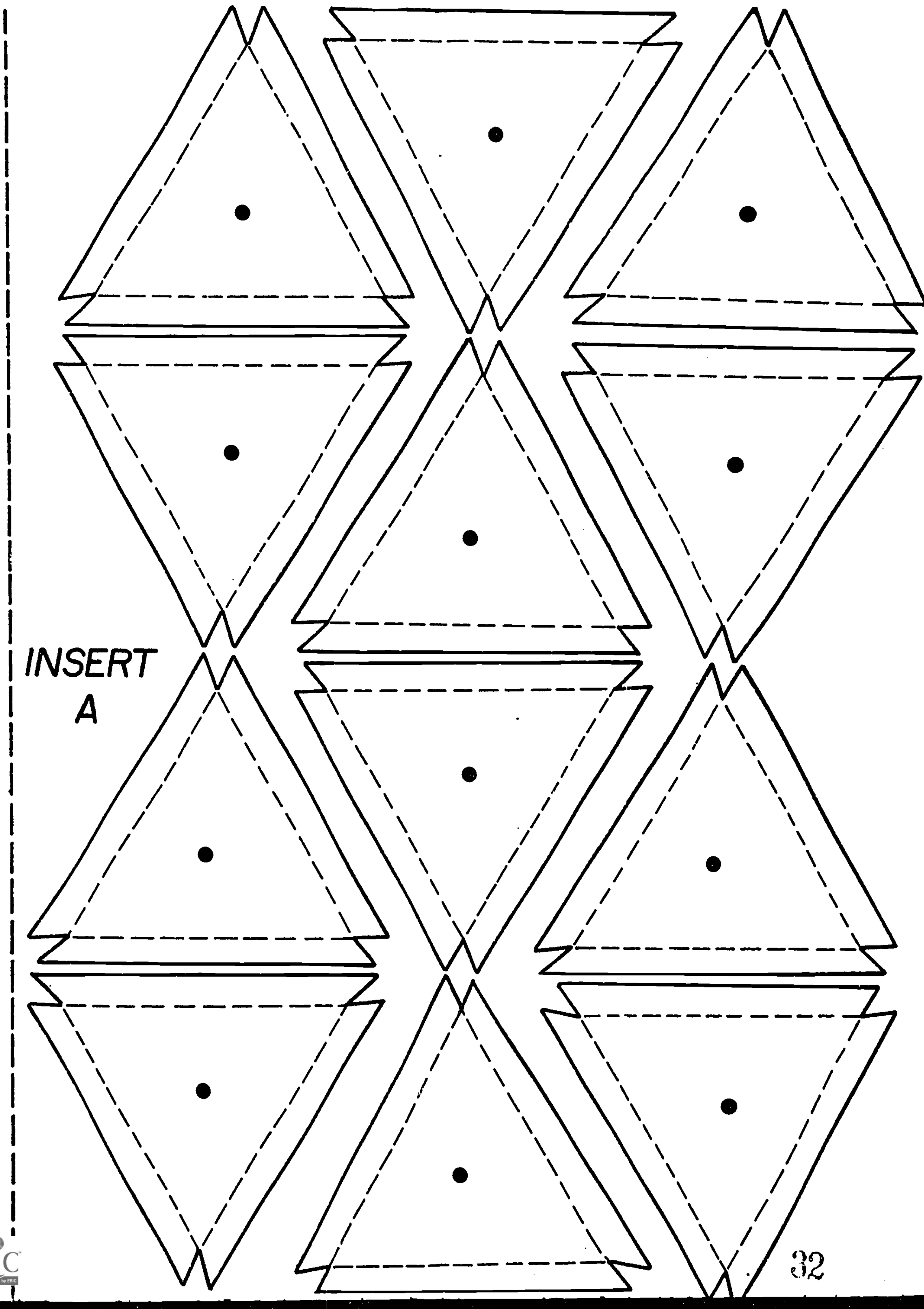


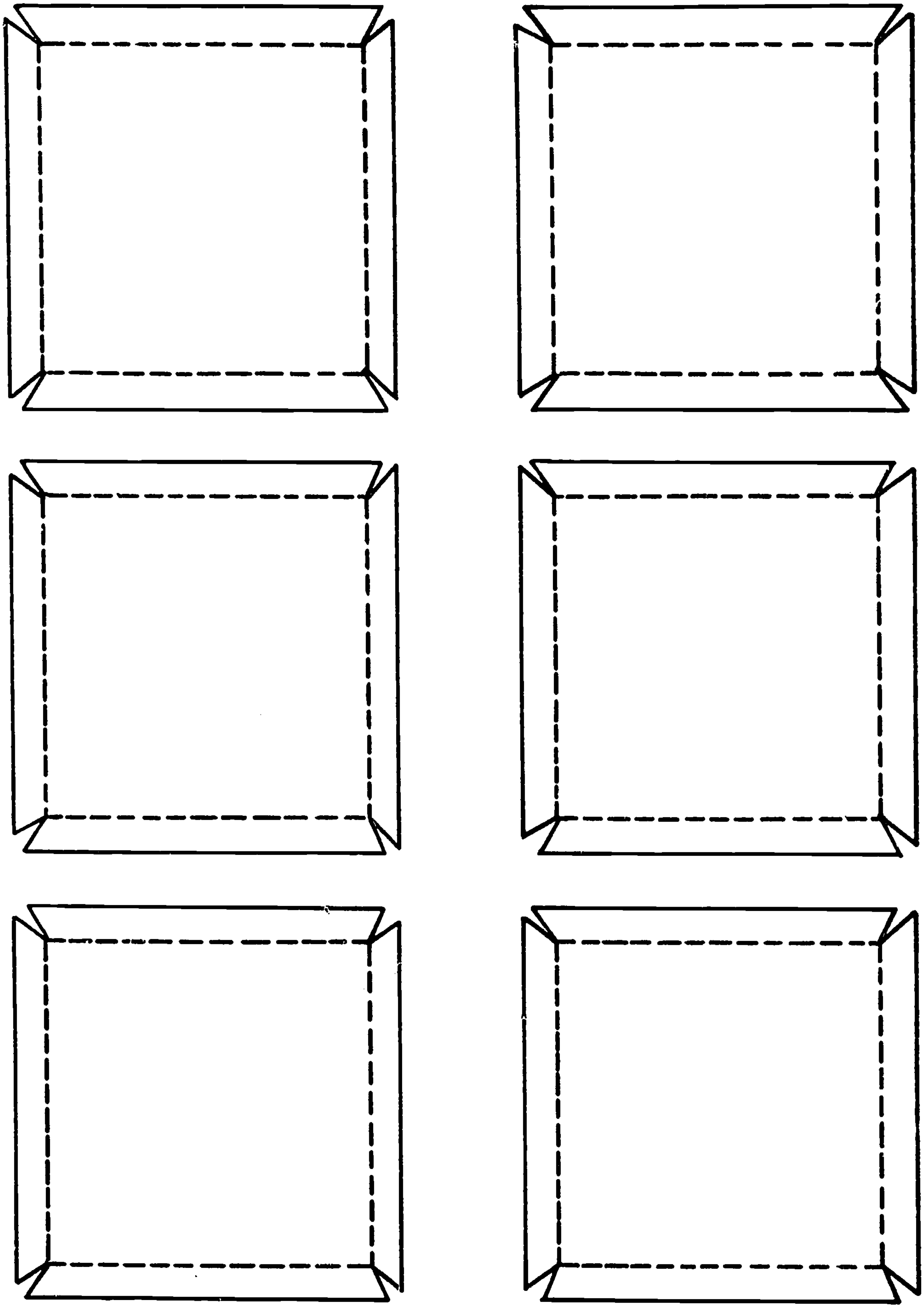
b. Connect the flaps of adjacent panels with a rubber band. (1 to 4; 2 to 4; 3 to 4.)

c. Fold the pattern into the shape of a tetrahedron. Connect the other three edges (a, b, c) with rubber bands.



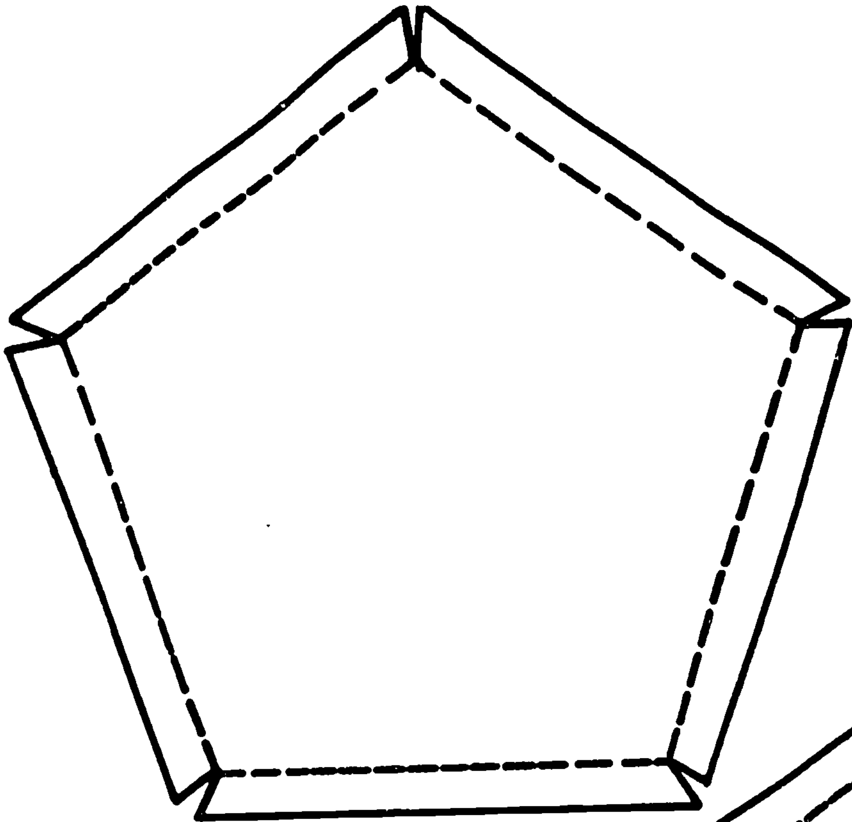
Follow the same procedure with each of the other patterns you have already formed and build models of a cube, octahedron (bottom of Traffic Control Tower), icosahedron (Nuclear Power Chamber) and dodecahedron (Cloud Shooting Station). Don't forget to have one person hold the pattern down while the other two apply the rubber bands.

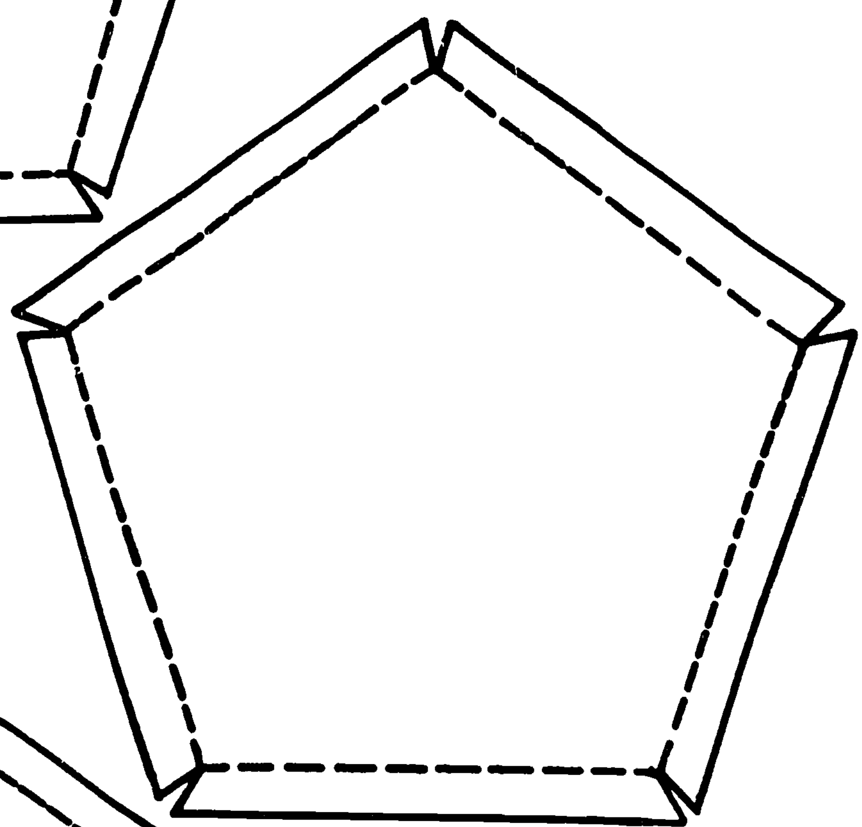

INSERT
A

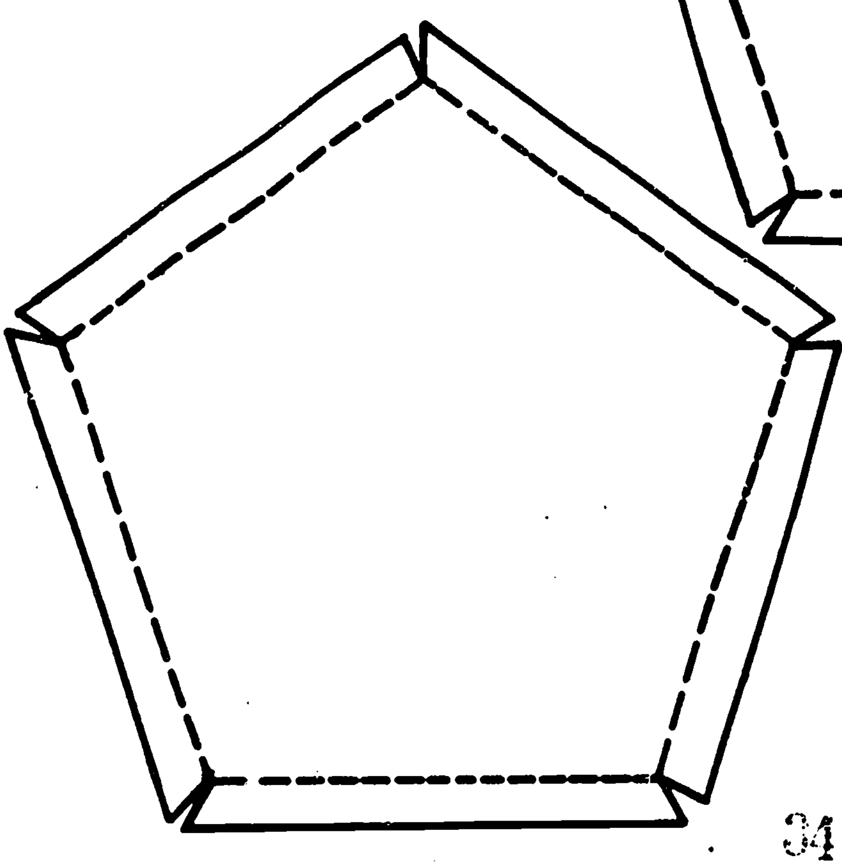


INSERT B

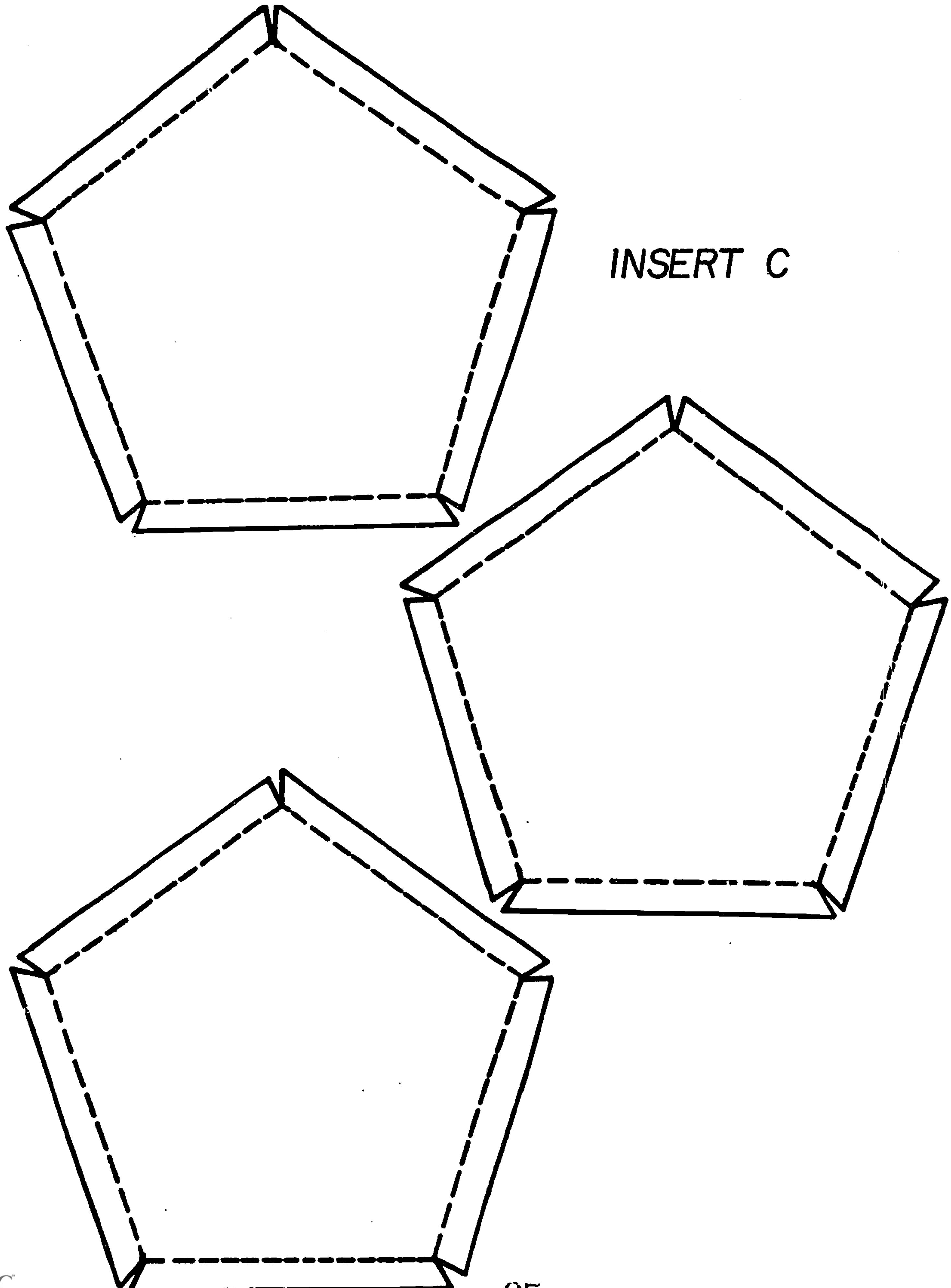
INSERT C



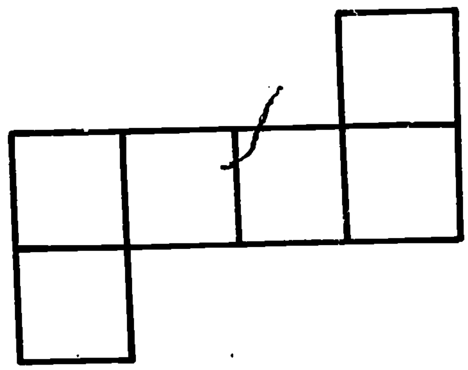




INSERT C



3. To build a model of the cube you started with this pattern:



(a) How many different two-dimensional patterns do you think there are which can be folded to form a cube?

(Guess!) _____

(b) See how many patterns you can find which can be folded to form a cube.

HINTS:

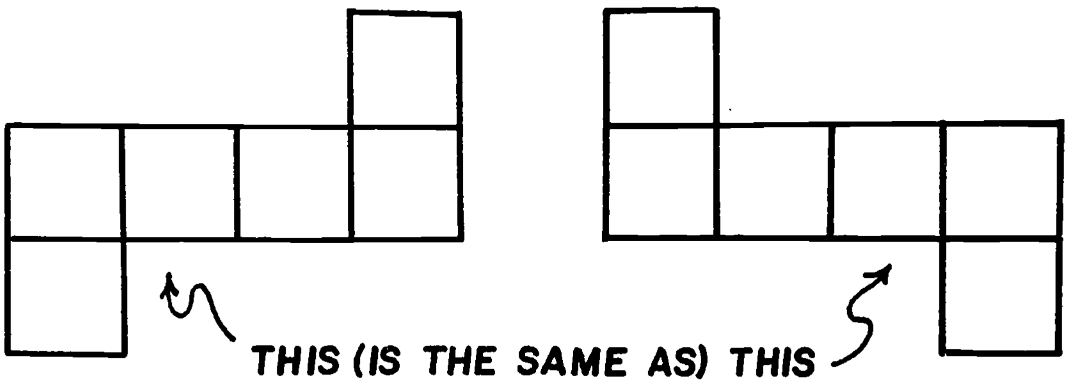
(1.) Take your original model; remove any seven rubber bands (no more than 3 from the edges of the same face). Flatten the panels out on your desk. Is this pattern different?

(2.) Now remove just one panel from the pattern you came up with. Move it around. Connect it in another place. Can this pattern be folded to form a cube?

(3.) Keep moving one or more panels around and record all the different patterns you are sure can be folded to form a cube. (Use insert AA to record your patterns. The original has already been recorded for you.)

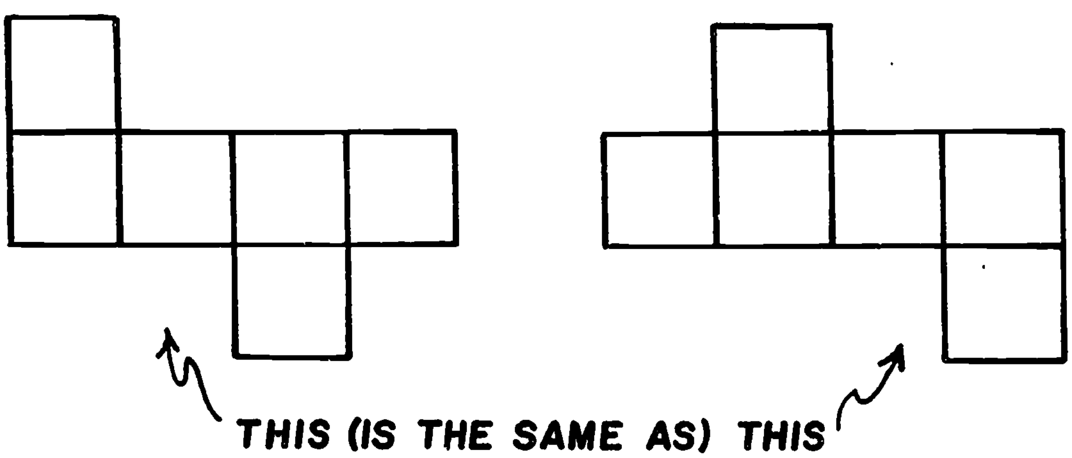
CAUTION:

Make sure each pattern you record is different.



These two patterns are the same. The first can be flipped to get the second.

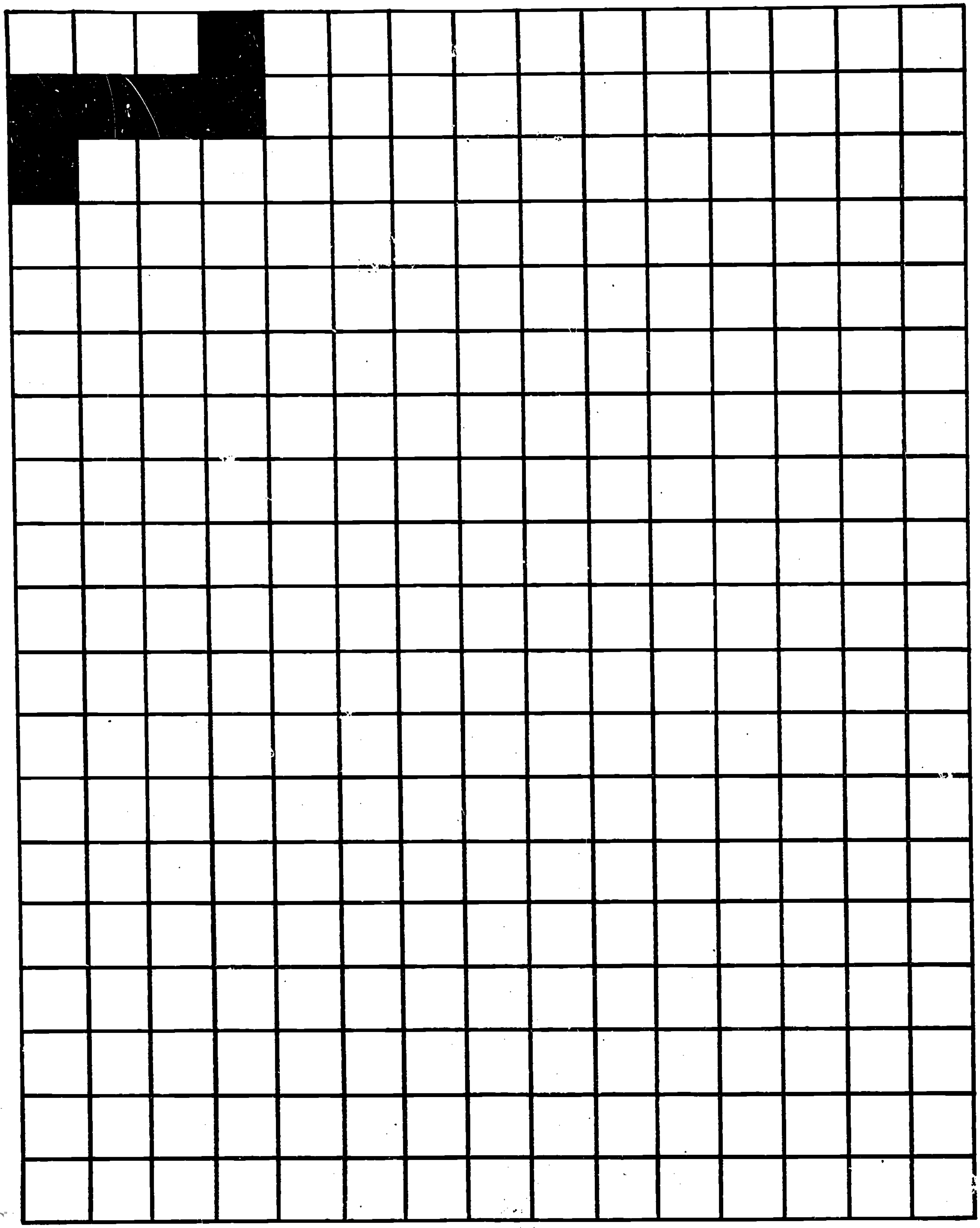
These two are also the same:



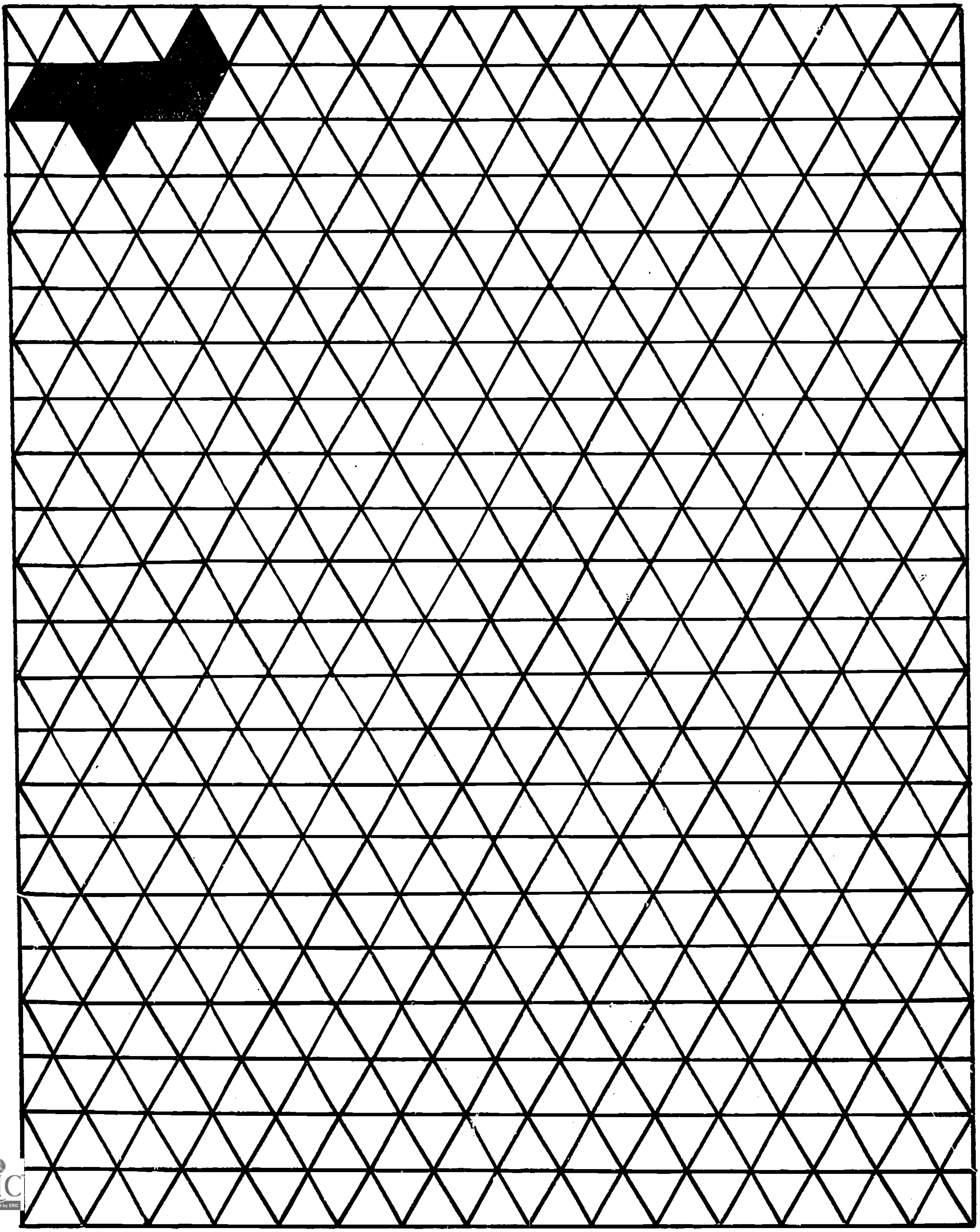
The first has been rotated to get the second.

REMEMBER — no flips or rotations.

INSERT A-A



INSERT B-B



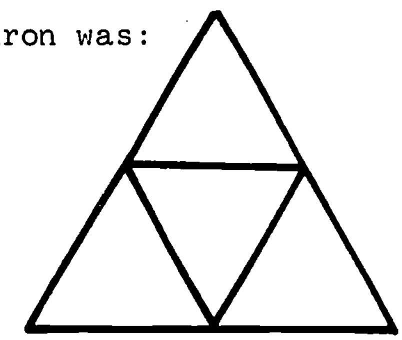
4. See how many different patterns you can find for the octahedron. Record them on the triangular grid provided (insert BB). The original has already been recorded.

   No flips or rotations — please!

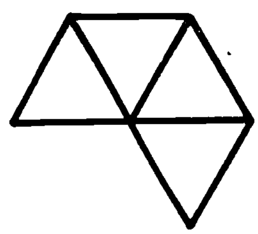
ON YOUR OWN

(Try to do the following without the aid of the panels and rubber bands. Fold the patterns in your mind.)

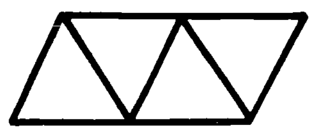
1. The pattern we used for the tetrahedron was:



   Could we have used this pattern?



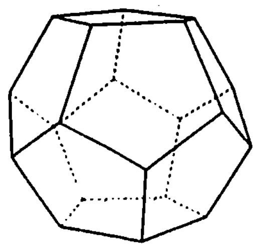
   How about this one?



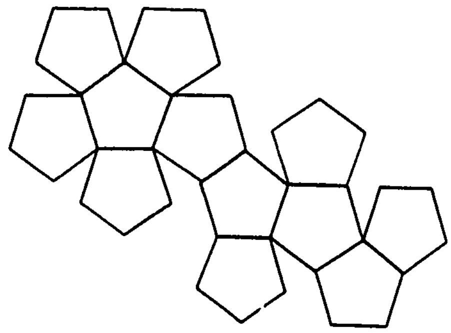
   Are there any other patterns which can be used for the tetrahedron? If so, how many?

2. For each of the following identify the patterns (from those shown) which you think can be folded to form the given space figure.

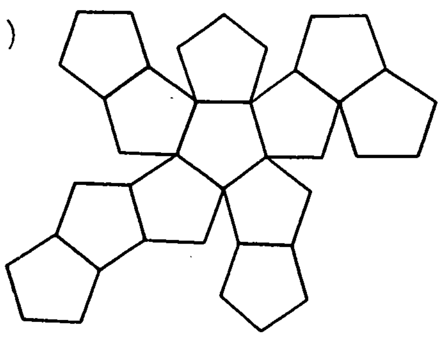
a.



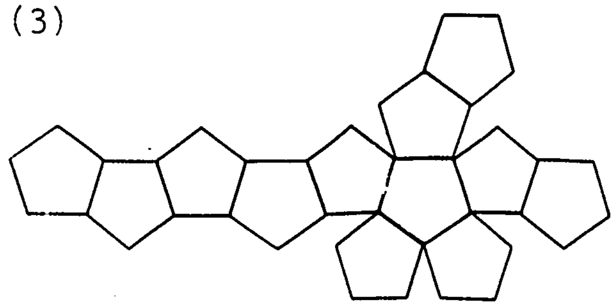
(1)



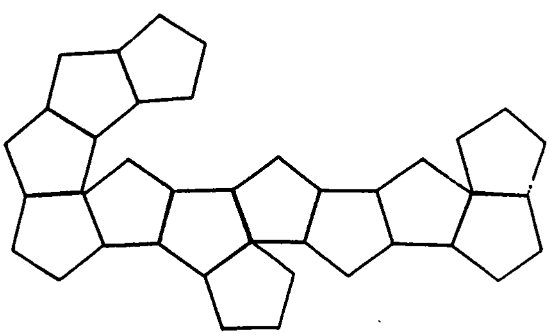
(2)



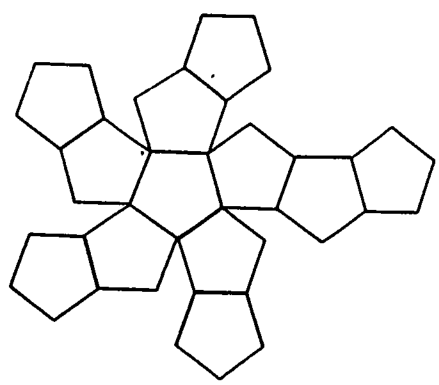
(3)

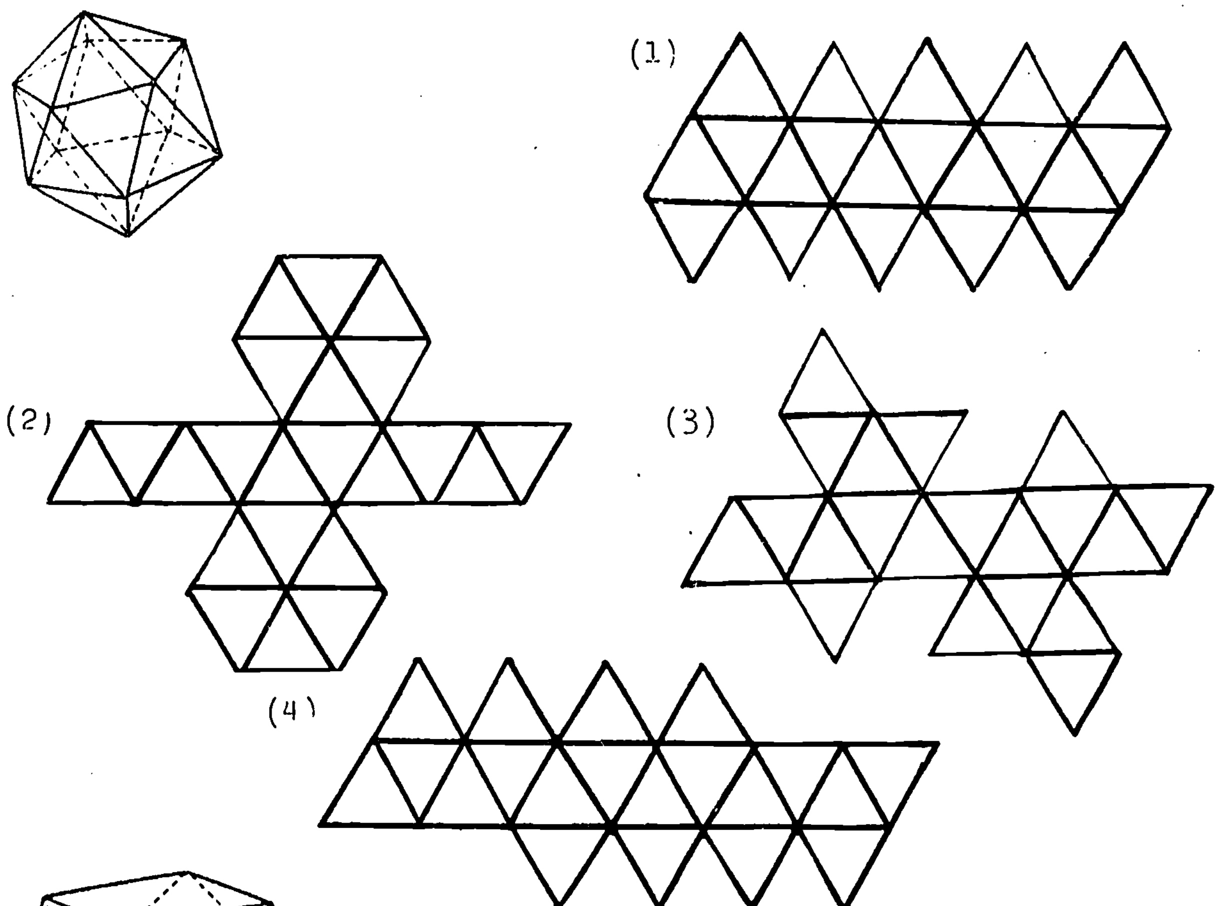


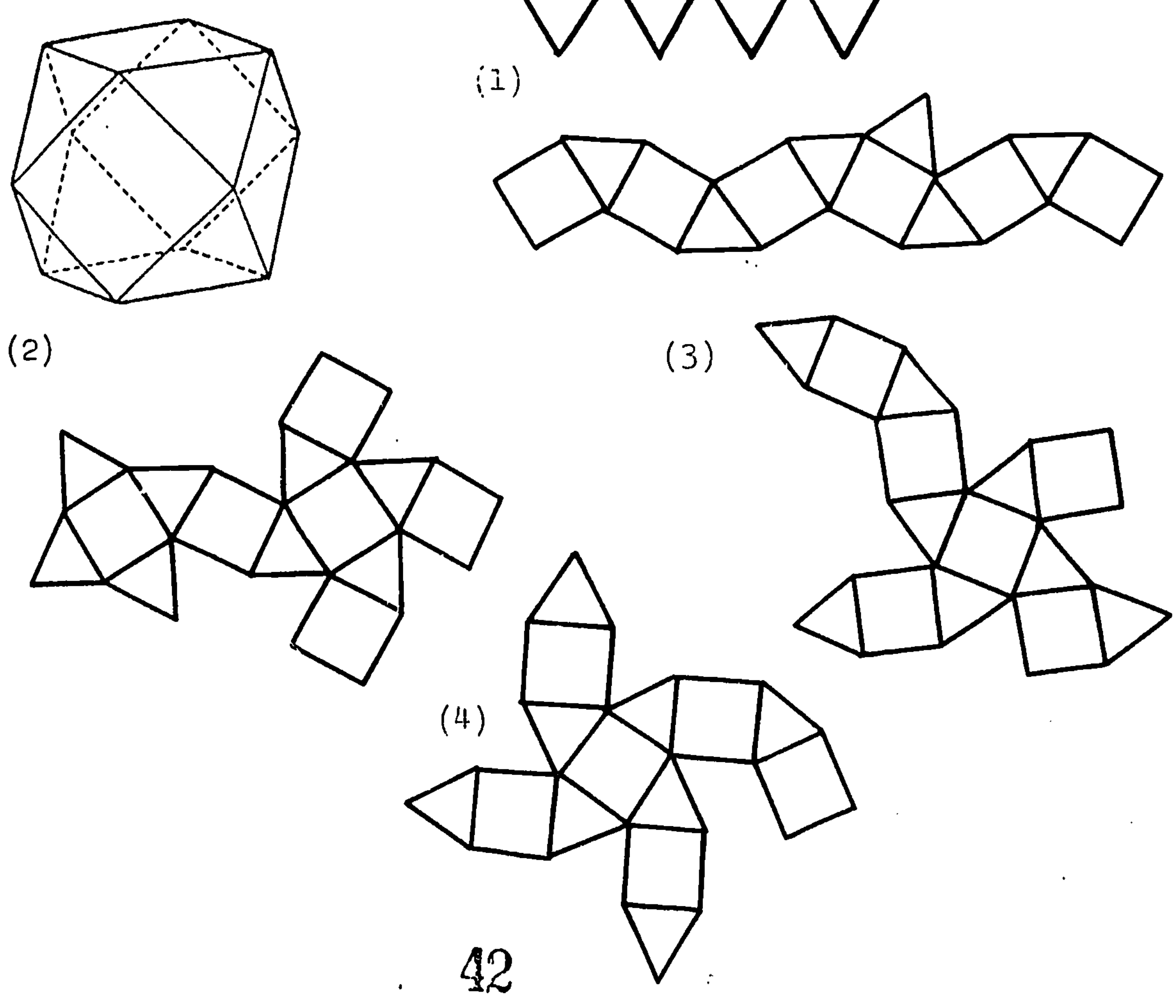
(4)



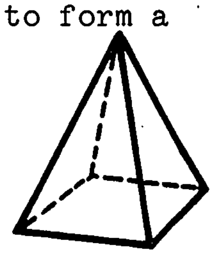
(5)



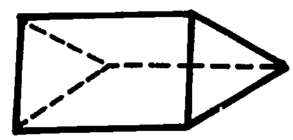

(1)
(2)
(3)
(4)


(1)
(2)
(3)
(4)

3. Sketch three patterns which can be folded to form a square pyramid.



4. Sketch three patterns which can be folded to form a triangular prism.



TAKING A NUMBER BREAK

Circle the correct response for each of the following.

1. In finding the answer to 25 × 73 the steps could be written as

	73
	× 25
Step 1	365
Step 2	146
Step 3	1825

In step 2, 146 is placed where it is to stand for

(a) 2 × 73 (b) (1 × 100) + (4 × 10) + 6
(c) 70 × 25 (d) 20 × 73

2. A student has made the same mistake in each of the following subtraction problems.

$$\begin{array}{r} 57 \\ -\underline{\;\;8} \\ 59 \end{array} \qquad \begin{array}{r} 203 \\ -\underline{\;25} \\ 188 \end{array} \qquad \begin{array}{r} 365 \\ -\underline{\;87} \\ 288 \end{array} \qquad \begin{array}{r} 518 \\ -\underline{336} \\ 282 \end{array}$$

If he makes the same mistake in the problem $\begin{array}{r} 196 \\ -\underline{\;78} \end{array}$

his answer will be

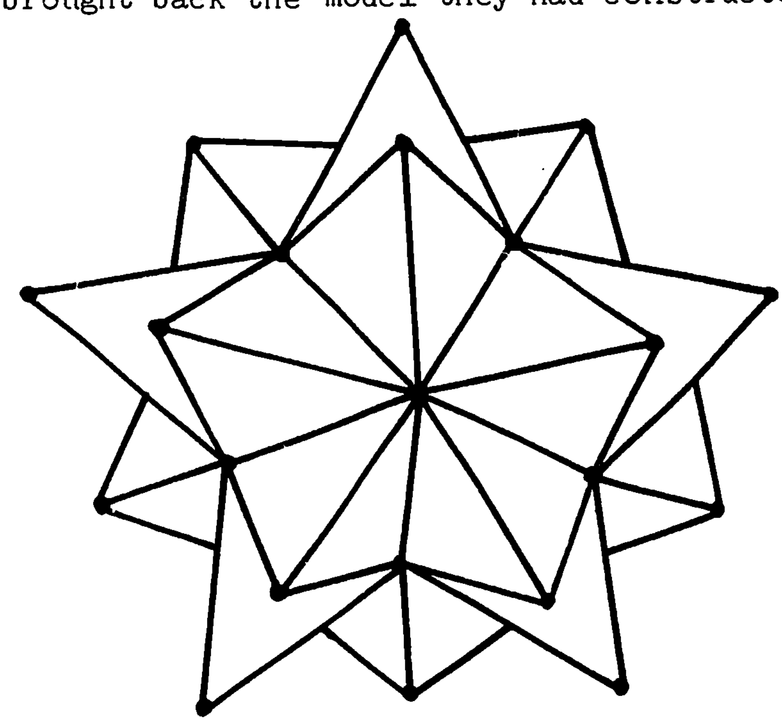
(a) 122 (b) 128 (c) 118 (d) 274

STRINGING STARS

SPRING FORMAL
DECORATION COMMITTEE MEETING
3:30 ROOM 105

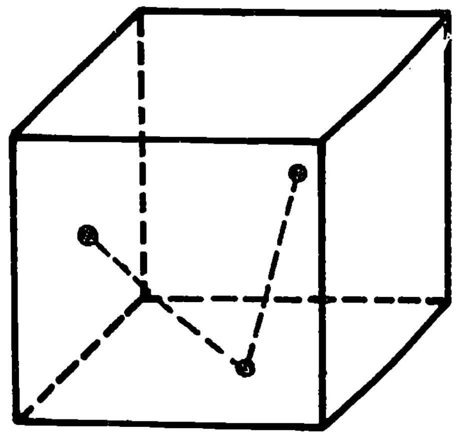
The decorations committee got together to discuss plans for decorating the gym for the Spring Formal. An outer space theme had been selected and the committee was trying to come up with appropriate props.

Judy suggested constructing a huge star polyhedron to hang from the ceiling at the center of the gym. "What's a star polyhedron?" someone asked. Judy ran down to the math room and brought back the model they had constructed from straws.



Everybody liked the idea. They decided to make it out of styrofoam strips and sequins.

The committee wanted to use some miniature blinking lights to represent stars. They couldn't decide where to string them. Dan suggested placing a hook at the middle of each wall, the floor and the ceiling. He thought the lights could be strung from the hook on a surface to the hook on each of its adjoining surfaces. Dan explained that adjoining surfaces had an edge in common and drew this picture on the board.



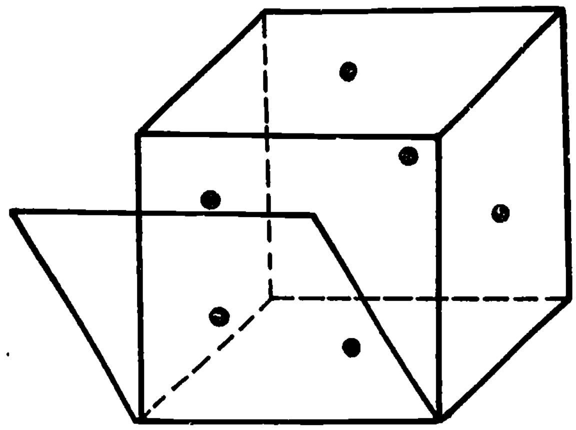
Before the group had a chance to discuss Dan's suggestion the bell for the late bus rang and the group hurried out.

CLASS ACTIVITY

1. You and two of your classmates are members of the decorations committee. The next day in math class you wonder about Dan's suggestion. The gym is cubical. If you string the lights as Dan suggested, what figure would you get?

    A. Use your model of a cube to represent the gym. Take the cube apart and draw the diagonals of each face. The point at which the diagonals meet will be called the middle point. (It is the same distance from each vertex.) Punch a hole at each of the middle points.

B. Put the cube back together; leave one face open.



C. Use yarn or pipe cleaners to represent the lights. Connect the middle points of adjacent faces with the yarn or pipe cleaners until you can recognize the figure being formed.

What figure will be formed if the middle points of adjacent surfaces in a cubical gym are connected with strings of blinking lights?

____

2. Suppose the dance is going to be held in a new ultra-modern ballroom shaped like an octahedron. The committee still wants to string lights to represent stars. What will happen if you carry out Dan's suggestion in this situation?

   A. Use your model of an octahedron to represent the room. The middle points have already been marked on the triangular panels. Punch a hole at each middle point of the faces of your octahedron.
   B. Leave one face of the octahedron open.
   C. Repeat step C of Activity 1.

What figure will be formed if the middle points of all adjacent surfaces in an octahedron-shaped room are connected with strings of blinking lights?

____

ON YOUR OWN

1. What figure was formed by connecting the middle points (of adjacent faces) of a cube? __________

2. What figure was formed by connecting the middle points of an octahedron? __________

3. What figure would be formed by connecting the middle points of a tetrahedron? __________

4. Two Platonic solids are called duals if one has the same number of vertices as the other has faces and vice versa. Which Platonic solid is its own dual? __________

5. What is the dual of the cube? __________

6. Without actually connecting them, identify the kind of figure that would be formed by connecting the middle points of an icosahedron.

   ____

7. What kind of figure do you think would be formed by connecting the middle points of a dodecahedron?

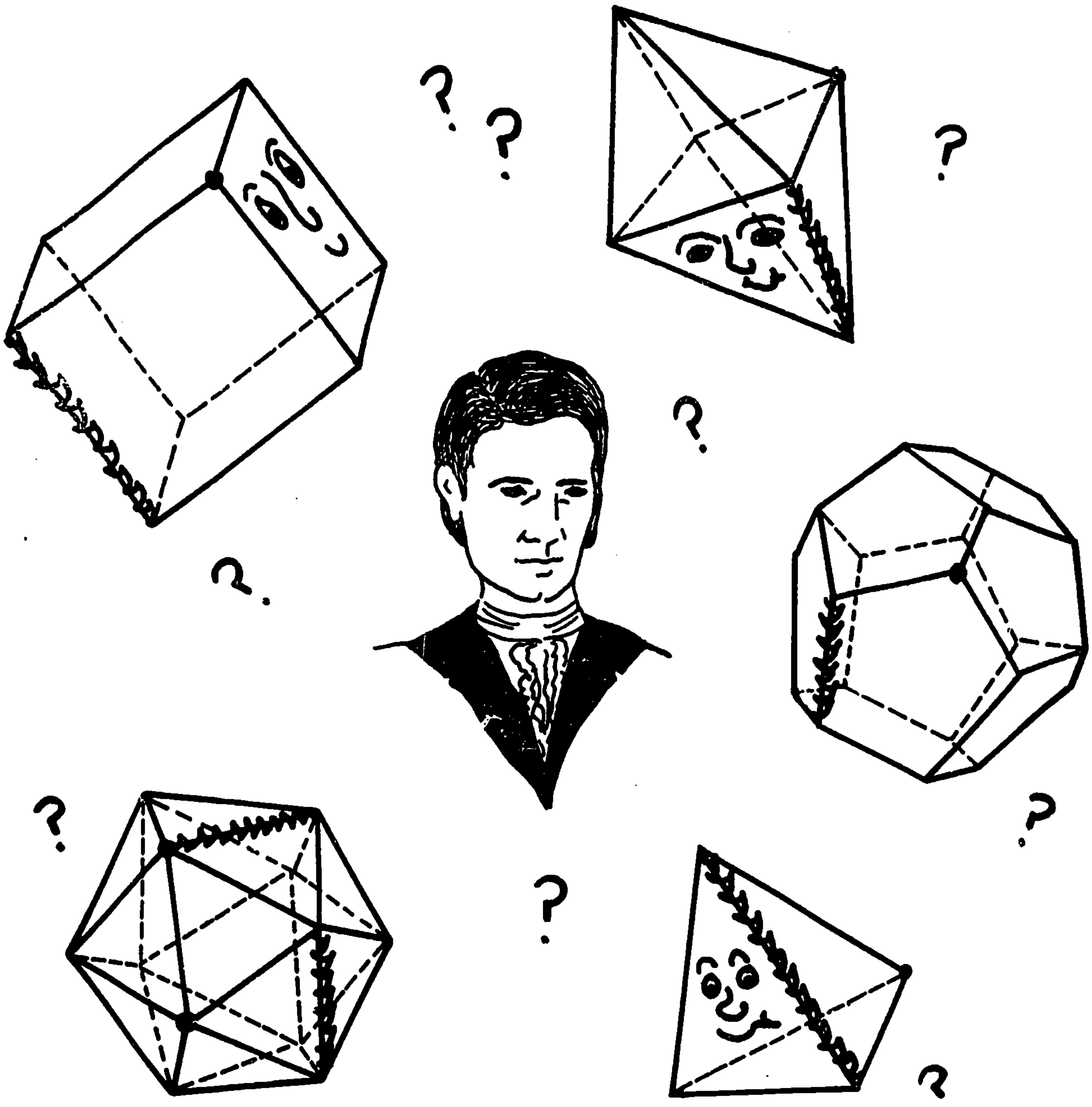
   ____

8. What is the dual of the dodecahedron? __________

9. Is there a figure which is the dual of a square pyramid? If so, what is it? __________

FACES, EDGES, VERTICES - DO THEY ADD UP?

In the 18th century, Leonard Euler, a Swiss mathematician, discovered a relationship between the number of faces, vertices and edges of a polyhedron.



Fill in the table on the next page and see if you can discover the same relationship that Euler found.

STRUCTURE	NUMBER OF FACES	NUMBER OF VERTICES	NUMBER OF EDGES
1. Bottom of the Traffic Control Tower (Octahedron)			
2. Top of the Traffic Control Tower (Tetrahedron)			
3. Electronic Servant Rental (Cube) (outer structure)			
4. Cloud Shooting Station (Dodecahedron)			
5. Nuclear Power Chamber (Icosahedron)			
6. Geopolis Hilton (Cuboctahedron)			
7. Middle structure of Under H_2O Research Center (Hexagonal Prism)			
8. Geopolis State University (Star polyhedron)			

EXERCISES

1. For the tetrahedron:

   number of faces + number of vertices = __________
   number of edges = __________

2. For the octahedron:

   number of faces + number of vertices = __________
   number of edges = __________

3. For the dodecahedron:

   number of faces + number of vertices = __________
   number of edges = __________

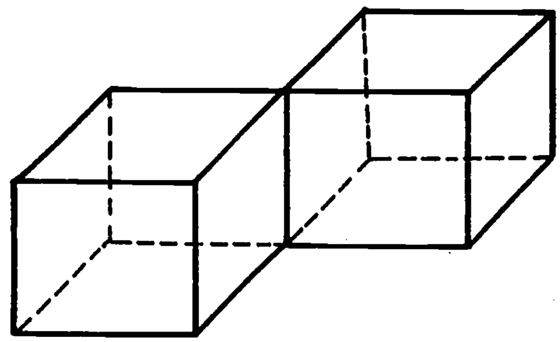
4. Write a sentence which expresses the relationship between the number of faces, vertices and edges for the figures in the above exercises. [Hint: Study the two answers that you have recorded in each exercise above. If that doesn't provide a clue, compare the numbers in the first two columns (Faces, Vertices) of the table with the numbers in the third column (Edges).]

5. Which of the other figures listed in the table follow the same pattern?

6. A certain polyhedron has 12 vertices and 5 faces. How many edges does it have?

7. A certain polyhedron has 500 vertices and 1000 edges. How many faces does it have?

8. Consider the following figure formed by two cubes which share an edge:



number of faces = ________

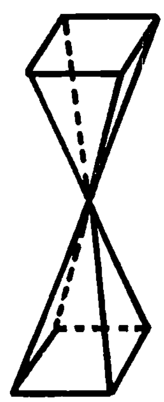
number of edges = ________

number of vertices = ________

F + V = E + ____

Is the result different for this figure? ____ If so, why?

9. Consider the following figure formed by two square pyramids which share a vertex:



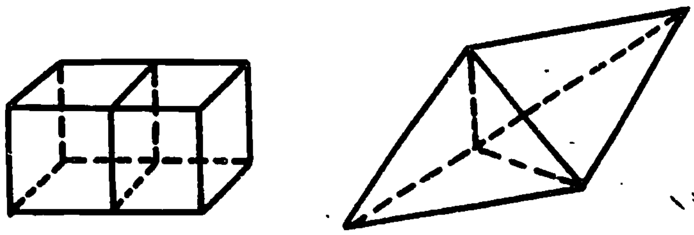
number of faces = ________

number of vertices = ________

number of edges = ________

Does Euler's formula (F + V = E + 2) hold for this figure? Why or why not?

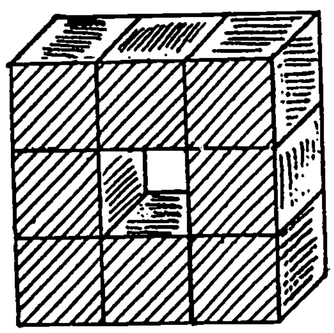
10. Consider the following figures formed by two regular polyhedra which share a face:



Do you think that Euler's formula will hold for these figures? ________

Explain Your Answer

11. Study the following figure which is made up of eight sugar cubes.

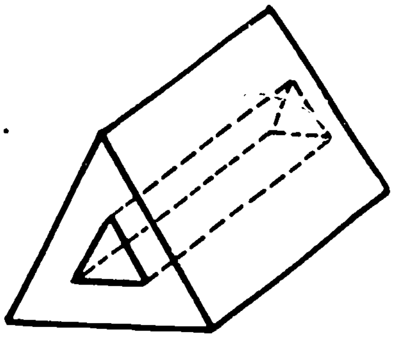


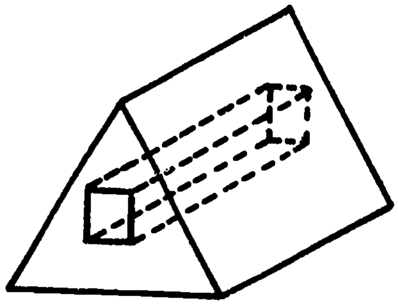
If you count all the faces, edges and vertices which are in the figure, does Euler's Formula hold? (Hint: There are 32 faces in the figure.)

faces = ____
vertices = ____
edges = ____

F + V = E + ____

12. The two triangular prisms pictured below have tunnels cut out of them. Figure a has a triangular tunnel; figure b has a rectangular tunnel.

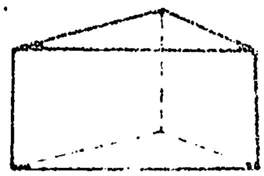
a. 

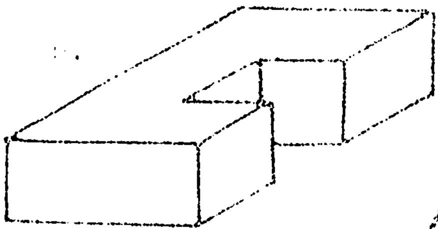
b. 

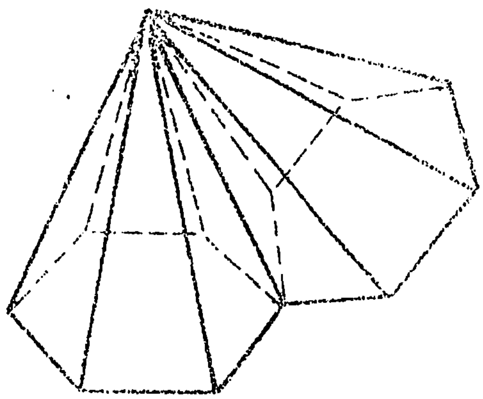
number of faces = ____	number of faces = ____
number of vertices = ____	number of vertices = ____
number of edges = ____	number of edges = ____
F + V = E + ____	F + V = E + ____

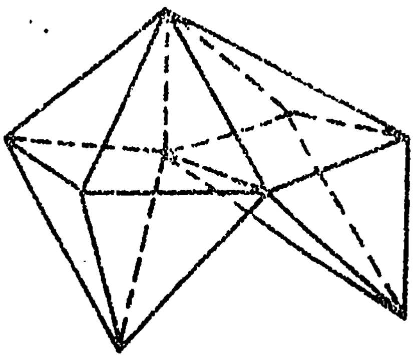
Do both figures satisfy Euler's Formula? ________ If not, why not?

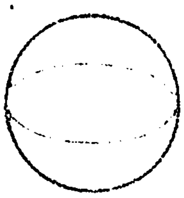
13. Which of the following figures do you think will satisfy Euler's Formula? __________ Try to answer without counting.

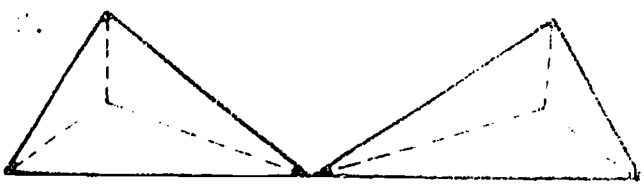






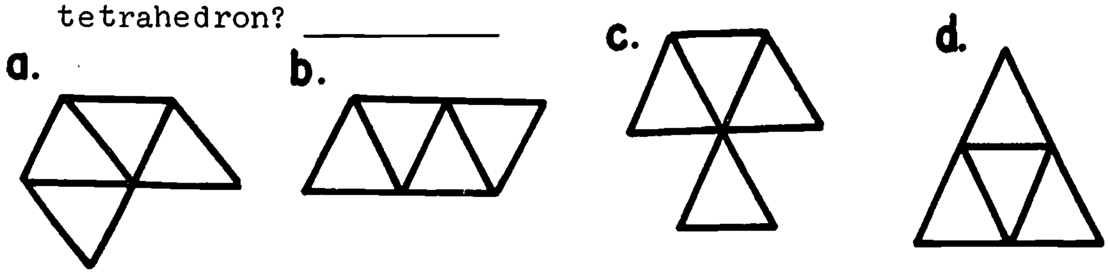






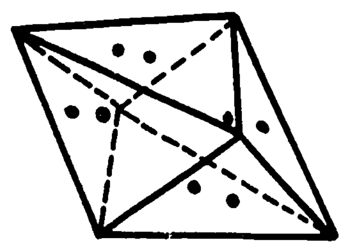
✔ POINT!

1. Which of the following patterns can be folded to form a tetrahedron? __________



2. Sketch three patterns which can be folded to form a cube.

3. Given the following octahedron with the middle points of each face indicated:



The figure formed by connecting the middle points of adjacent faces is a: ______________

a. cube. b. tetrahedron. c. octahedron. d. pyramid.

4. The dual of an octahedron is a: ______________

a. cube. b. tetrahedron. c. icosahedron. d. octahedron.

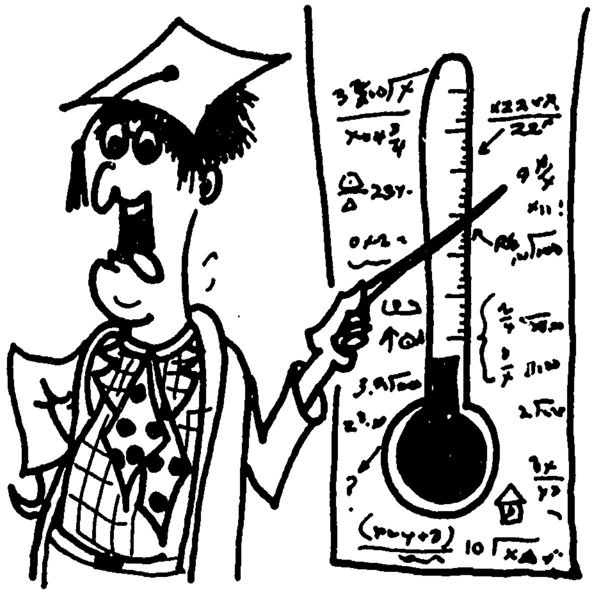
5. If a certain polyhedron has 11 faces and 15 vertices, it has _____ edges.

a. 26 b. 52 c. 28 d. 24

6. The smallest number of vertices a polyhedron can have is _____.

DOTS, CUBES AND DRAGSTERS

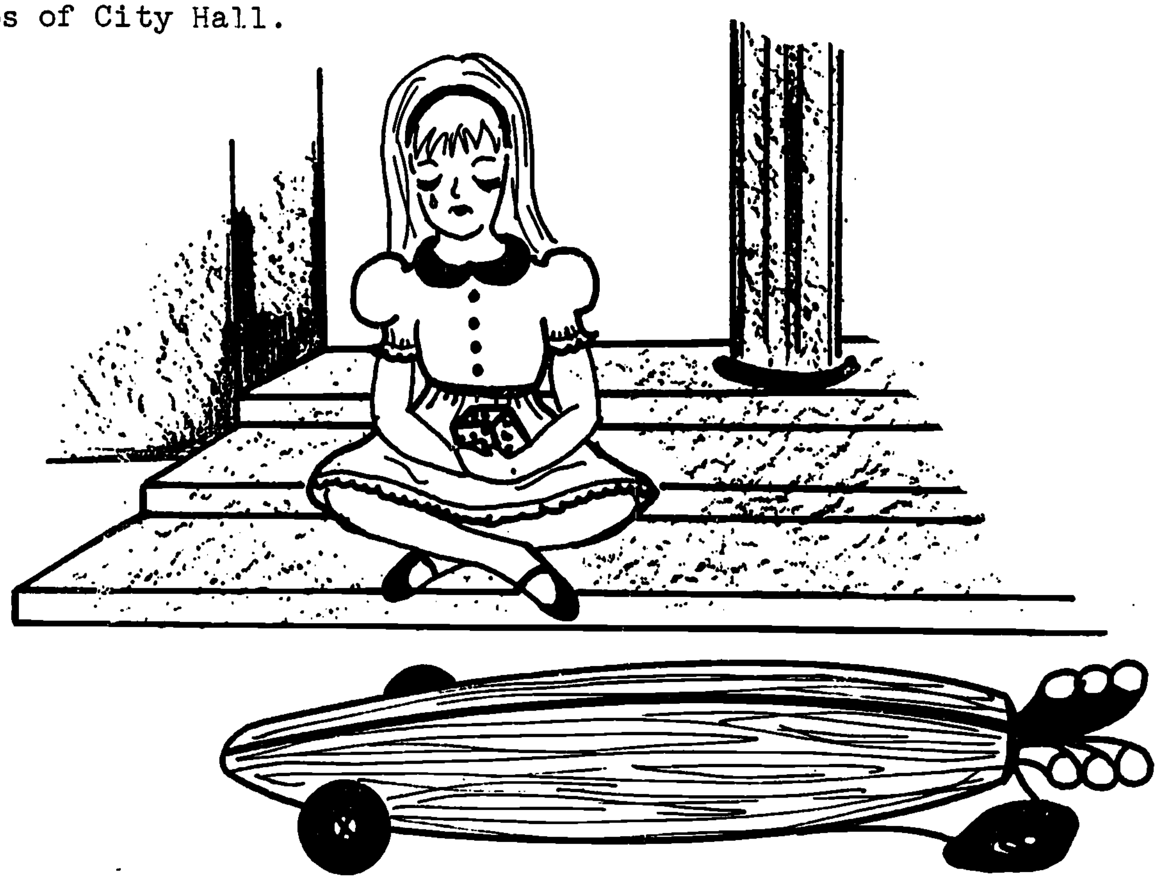
A call came into the Sugar Lake Police Station. The voice quivered, "It's been stolen; its's been stolen." Sargeant Sunday tried to calm the man and get the lowdown. Finally, the man identified himself. It was Professor Plat Formate. A cube-shaped package containing his secret formula, "TPS", had been lifted from his laboratory. If the cube-shaped structure made up of purple dotted sugar cubes containing "TPS" was not recovered within two hours the annual drag race at the Sugarflats would have to be cancelled. Sunday knew instantly that this was a case for the "Rod Squad."



The squad was contacted and within seconds their wheels were rolling and the pair began a search for the "TPS".

Meanwhile in a television press conference the Professor explained that there was enough "TPS" in the cube-shaped package for all nine of the heaps that were to run at the Flats. If the thief should use the entire package he would burn out his piston rings.

While the Professor was making his television debut, the "Rod Squad" received a tip concerning the possible whereabouts of the thief. They raced to the center of town where they spotted a small frail girl crying as she was sitting on the steps of City Hall.

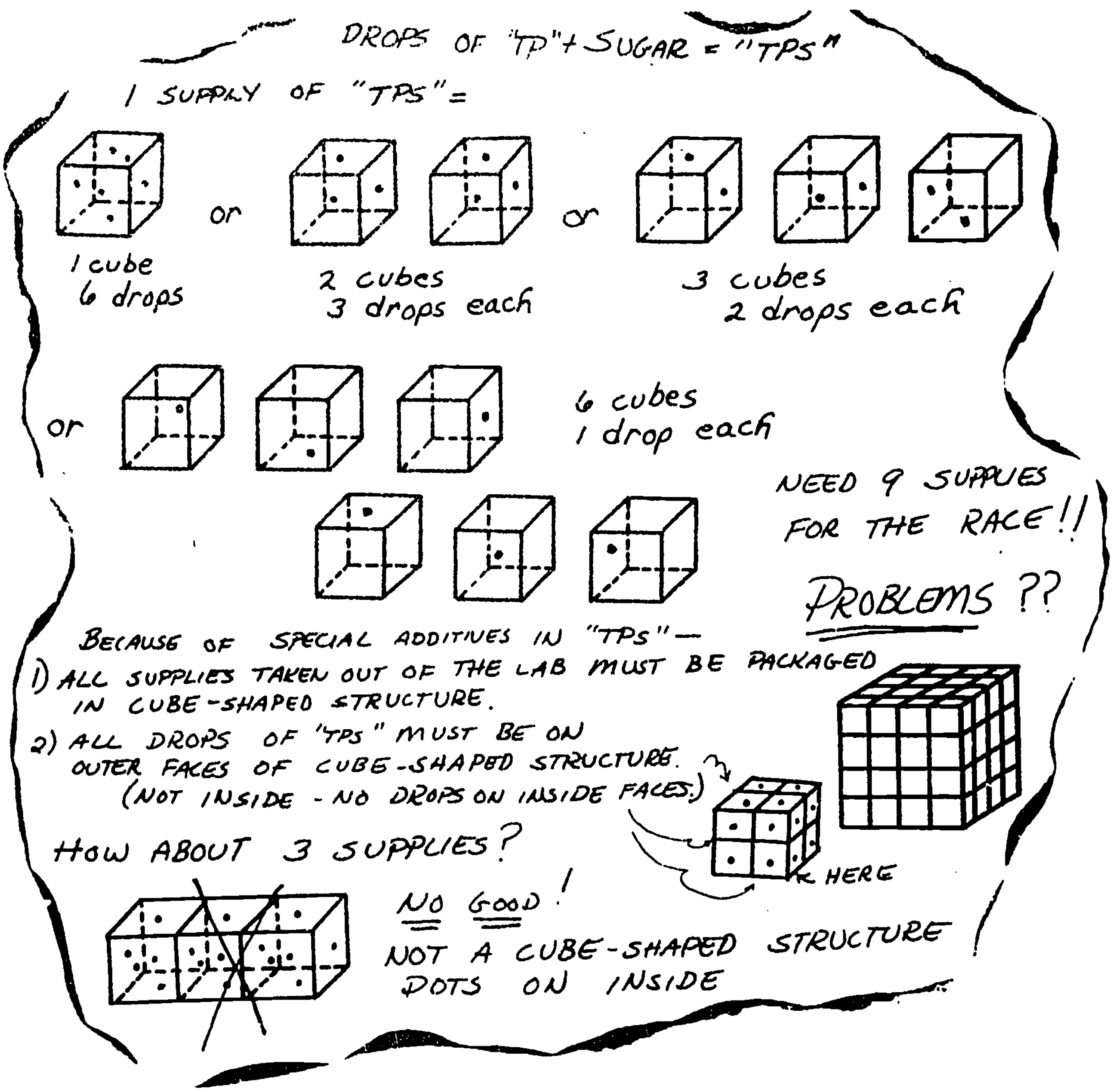


She was holding a cube-shaped package. One of the slicks on her motorized surfboard was as flat as the board. She was foiled in her attempt to have the fastest board on wheels. She surrendered the cube-shaped structure containing purple dotted sugar cubes, without any resistance.

The squad raced to the studio to pick up the Professor. As the groups burned down Main Street, on their way to the Flats, one of the "Rod Squad" asked Plat why he had stored the formula in the purple dotted cube-shaped package. Plat pulled a crumpled sheet of paper from his back pocket and passed it to him.

As you study Plat's scribbles keep these questions in mind:

1. What are the different ways that drops can be stored in sugar cubes to give one supply of "TPS"?
2. Why must the sugar cubes be packaged in a cube-shaped structure?
3. When the sugar cubes are arranged in a cube-shaped structure, must all the drops be visible?



The story you have just read is false (or at least we think it is). The names have been changed (just in case) to protect the innocent. Use the information from Plat's scribbles to work the following exercises.

EXERCISES

(Your teacher will give you a supply of sugar cubes to use in working the following exercises.)

1. If a single sugar cube with a drop on each face holds one supply of "TPS", why didn't the Professor simply package nine of these single cubes for the race?

2. If the sugar cubes were shaped into a $2 \times 2 \times 2$ cube-shaped structure (2 cubes on a side) with drops on each of the outer faces, how many cars could have received a supply of "TPS"?

   ____

3. Build a $3 \times 3 \times 3$ cube:
   a. How many cubes are needed? ___
   b. If a drop is placed on each of the outer faces, how many cubes contain:

      three drops? ___
      two drops? ___
      one drop? ___
   c. Do all the cubes which make up the $3 \times 3 \times 3$ cube have at least one drop?

4. How many supplies of "TPS" can be stored in a $3 \times 3 \times 3$ cube? ___

5. If the sugar cubes are packaged in $4 \times 4 \times 4$ cube,
   a. How many single cubes are needed to build it? _____
   b. How many cubes would have four drops? _____
      three drops? _____ one drop? _____
      two drops? _____ no drops? _____

6. In an $8 \times 8 \times 8$ cube-shaped structure how many cubes would have six drops? _____
   five drops? _____ three drops? _____
   two drops? _____ one drop? _____

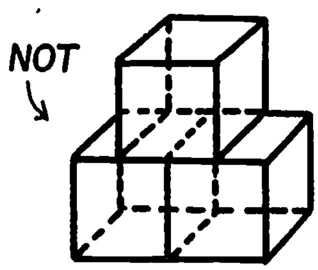
7. In a $100 \times 100 \times 100$ cube-shaped package how many cubes would have:
   four drops? _____ three drops? _____
   two drops? _____ one drop? _____

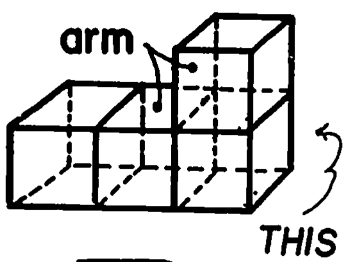
8. If nine supplies of "TPS" were needed for the race, the cube-shaped package stolen from the Professor's lab was a:
   a. $2 \times 2 \times 2$ cube b. $3 \times 3 \times 3$ cube c. $4 \times 4 \times 4$ cube
   d. $6 \times 6 \times 6$ cube e. $8 \times 8 \times 8$ cube f. $9 \times 9 \times 9$ cube

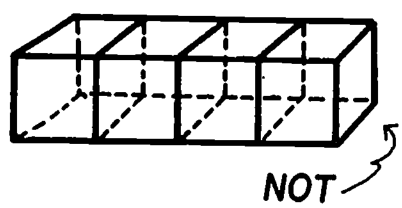
CLASS ACTIVITY

1. Use your cubes to form all possible <u>different</u> arrangements of not more than four cubes. In each arrangement the cubes should meet face on face; there should be at least one arm.

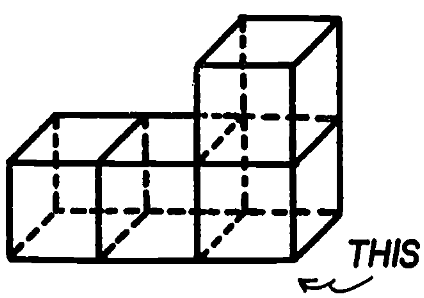
(meet face on face) (at least one arm)

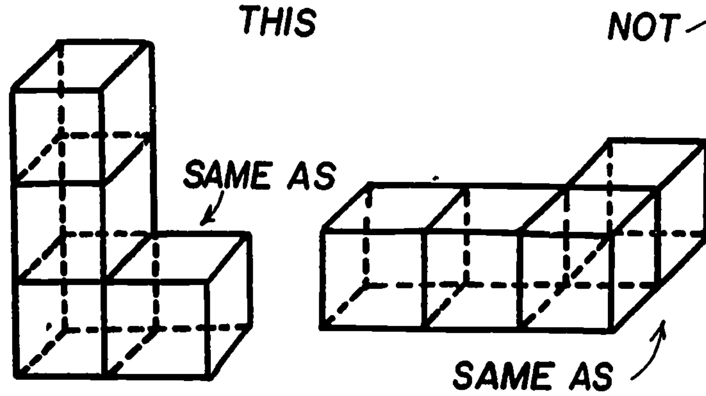






(different)





When you think you have found all of them, check with your teacher. (Hint, there are more than 5.)

2. Glue the cubes together.

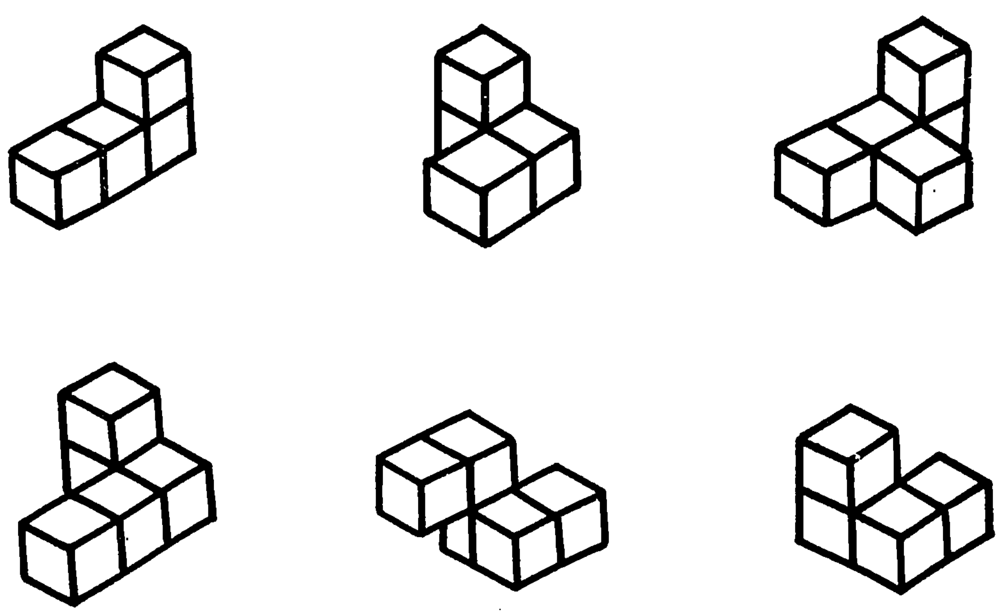
3. See if you can put the different pieces together in one $3 \times 3 \times 3$ cube.

4. Can you find another way to fit them together?

FOR THE PUZZLE ENTHUSIAST

If you enjoyed working the above puzzle and you have some patience left try this: (Get 27 more cubes from your teacher.)

1. Arrange the cubes into six pieces.

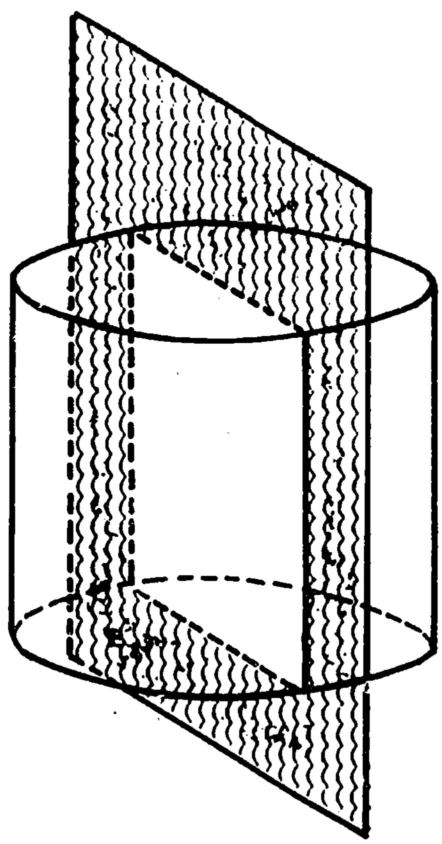


2. Build a $3 \times 3 \times 3$ cube with these six pieces.

SLICING THROUGH SOLIDS

The chief construction engineer of the new cylindrical-shaped computer building at Apollo Lift-Off Center is faced with a serious problem. A glass partition is to run through the middle of the building and extend from the roof to the ground floor. The computer will be housed in half of the building and kept in super-cold temperatures. Computer operators will work on the other side of the glass wall and keep a close watch on the electronic wizard.

This picture was drawn by the construction engineer to help him visualize what shape the wall must be.



It is important that he order a partition with the correct shape. A mistake would be costly.

THINK ABOUT IT

1. You are a rookie engineer on the construction job. You want to help your boss out. What shape wall would you suggest he order?

2. Suppose that the new computer turned out to be smaller than expected. The glass partition must still run from the roof to the ground floor but it will only cut off about one-third of the building. New plans must be sent to the glass company immediately. What will be the shape of this partition?
   Will this partition be the same size as the first one that was ordered?

3. If the building were shaped like a cone instead of a cylinder, what would be the shape of the glass partition (assuming that the partition runs from the tip of the cone to the base)?

The following experiments will help you see what happens when a two-dimensional surface passes through a three-dimensional solid. The glass partition in the computer building is a two-dimensional surface, and it represents part of what mathematicians call a <u>plane</u>. A plane can be thought of as a flat surface with no thickness, extending forever in two directions (length and width). Some common objects which represent parts of planes are the top of your desk, a flat sheet of paper, the surface of a calm lake, a thin sheet of ice, and the wall of your classroom. We will call flat two-dimensional surfaces <u>portions</u> of planes.

STATION I

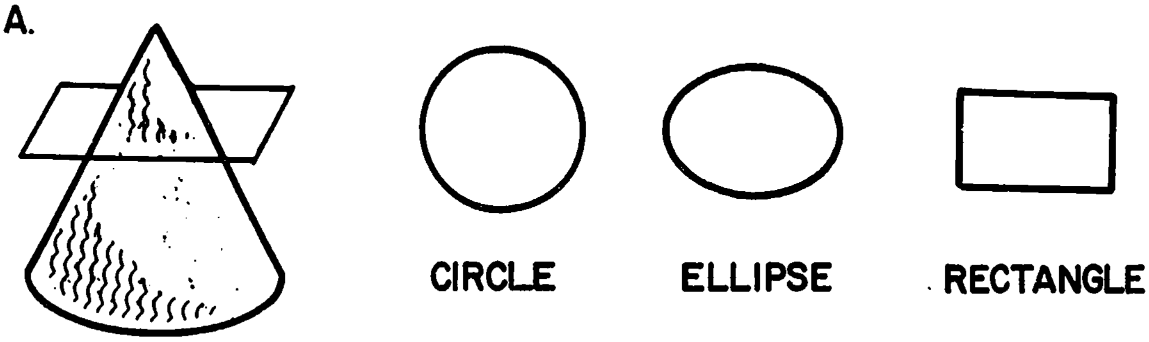
Materials; cone pattern, clay, wire, tape.

1. Fold cardboard pattern so that A is next to B. Tape the edges together.
2. Pack the clay into the cardboard cone, making sure it goes down the tip of the cone.
3. Release the pattern from the clay model.

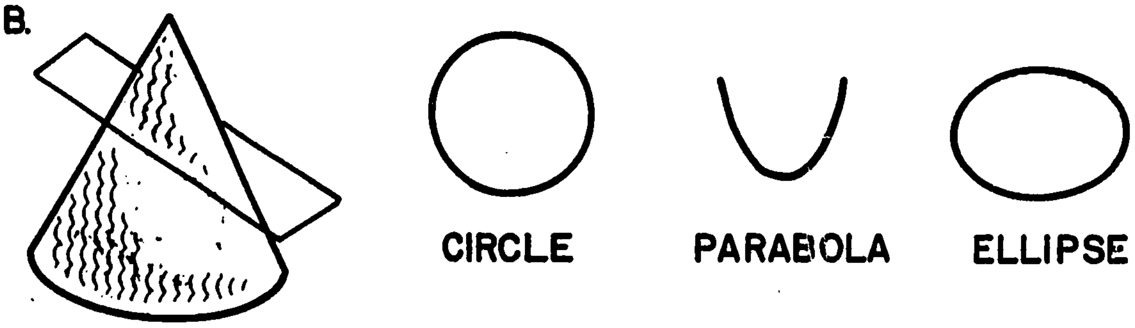
Use your wire to slice through the clay model in order to show the cut that would be made if a plane passed through the cone in each of the five different ways. Some possible shapes for each cut are shown. Circle the name of the shape which looks most like the one you get.

(Put the cone back together before making a new cut.)

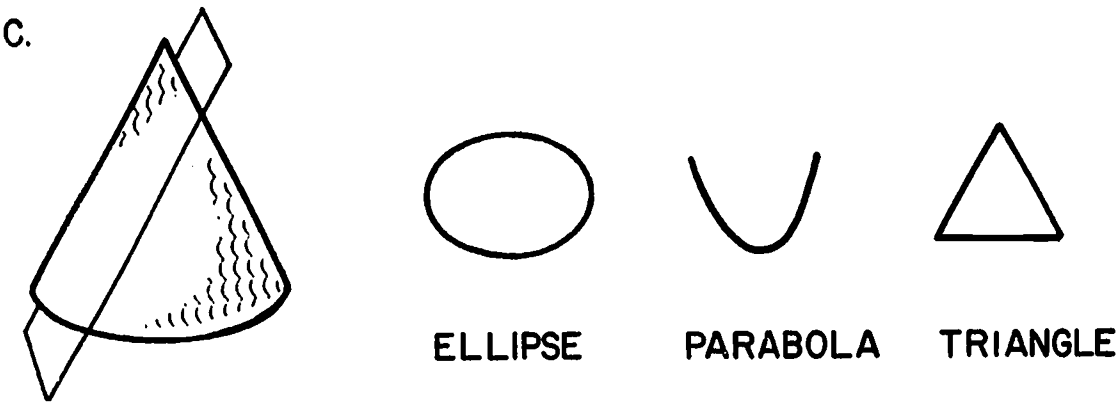
A.



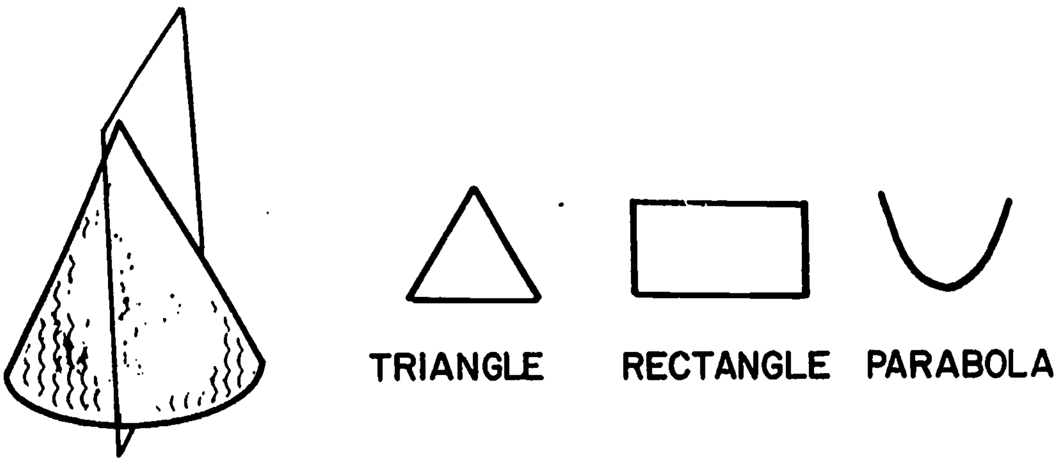
B.



C.

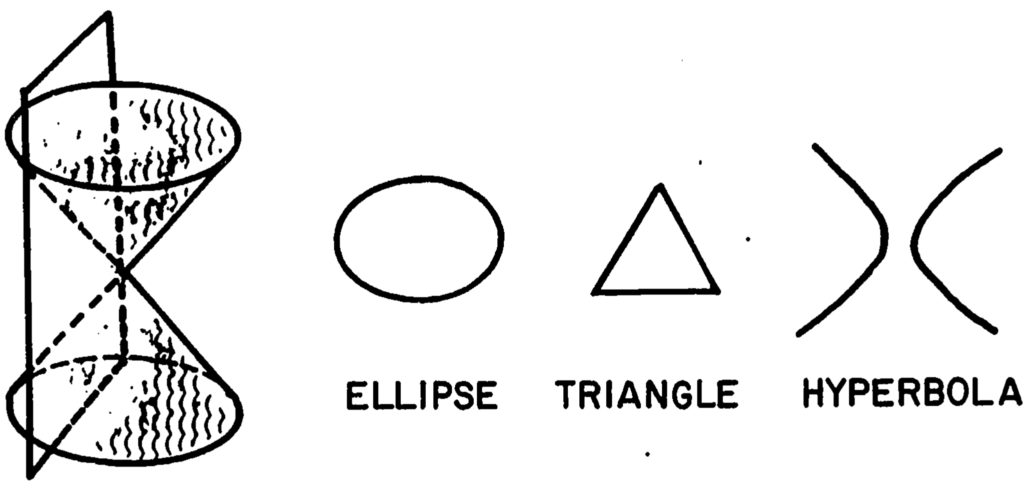


D.



What if you had two clay cones intersecting at a point with a plane passing through both of the cones?

E.



The circle, ellipse, parabola and hyperbola belong to a special set of plane figures called Conic Sections. These curves have some interesting properties which we will look at in Geometric Excursions II.

STATION 2

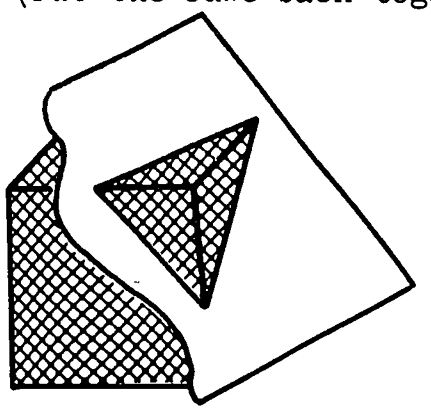
Materials: tagboard cube, clay, wire

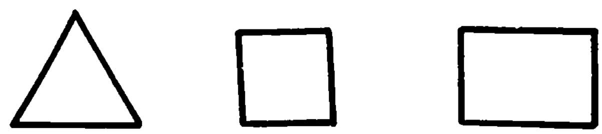
1. Pack clay into cube. (tightly)
2. Release tagboard from clay model.

Use your wire to slice through the clay model to show the cut that would be made if a plane passed through the cube in each of the five different ways. Some possible shapes for each cut are shown. Circle the name of the shape which looks most like the one you get.

(Put the cube back together before making a new cut.)

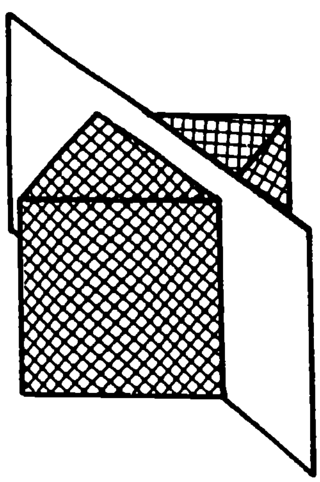
A.

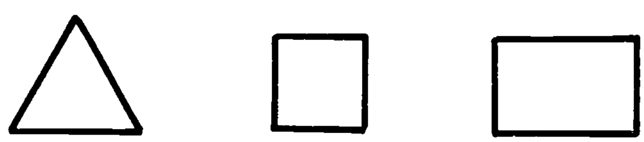




TRIANGLE SQUARE RECTANGLE

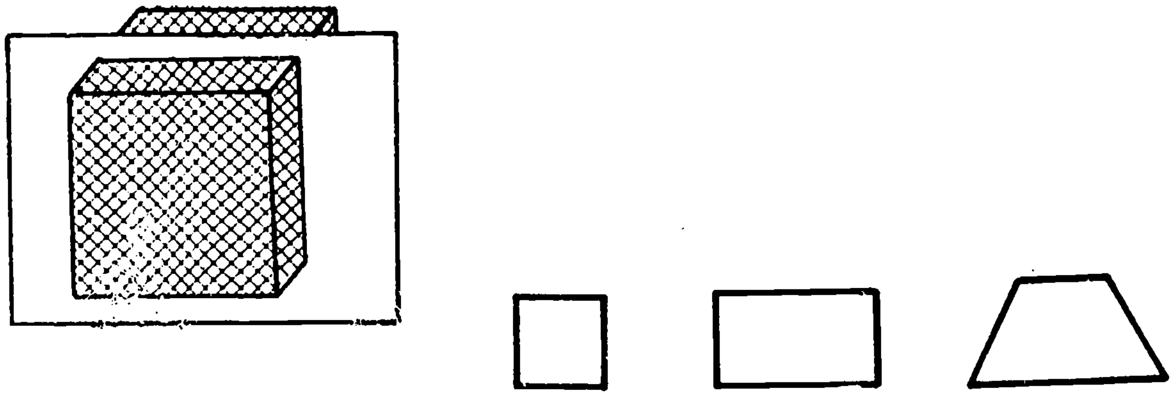
B.





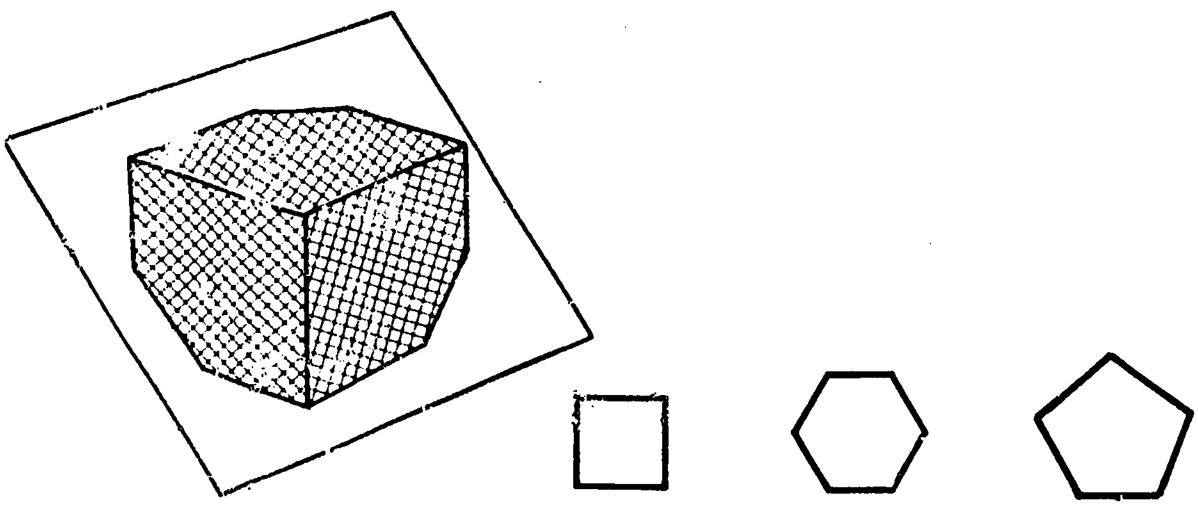
TRIANGLE SQUARE RECTANGLE

C.



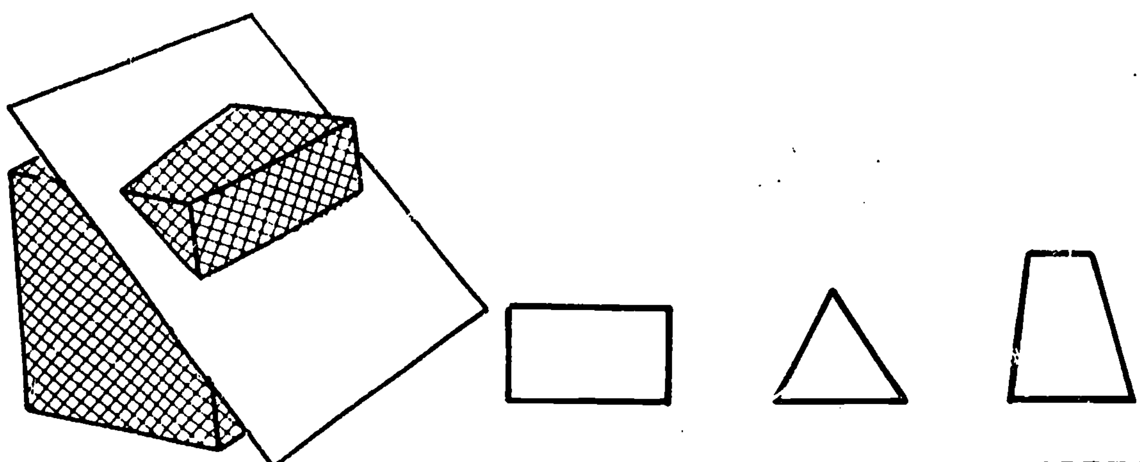
SQUARE RECTANGLE TRAPEZOID

D.



SQUARE HEXAGON PENTAGON

E.



RECTANGLE TRIANGLE TRAPEZOID

STATION 3

Materials: clay, wire

1. Roll the clay into a ball.
2. Slice through the clay model in at least four different ways. (Put the sphere back together before making a new slice.)

Sketch the shape of each cut.

1.

2.

3.

4.

What do all four shapes have in common?

Are all four shapes the same size?

What would be the shape of the largest possible cut? What would you have to do to get this cut?

STATION 4

Materials: clay, two paper cylinders, wire.

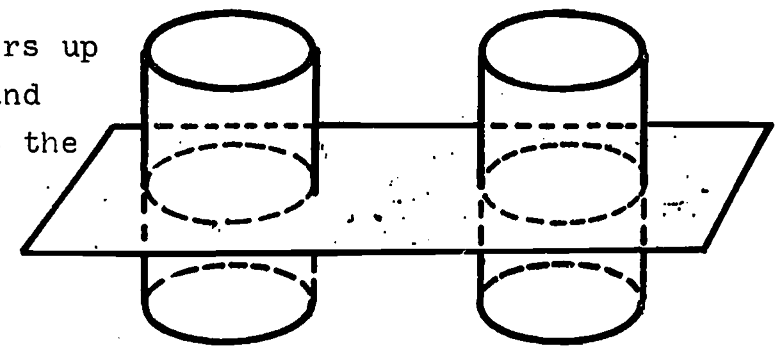
1. Pack clay tightly into both paper cylinders.

2. Release paper tubes from clay models. (Make sure the models are the same size.)

3. Trace the bottom and top face of each of the cylinders.

    What can you say about their shape and size?

4. Stand the two cylinders up next to one another and slice through both at the same time.



How do the plane sections that you get compare with each other in size and shape?

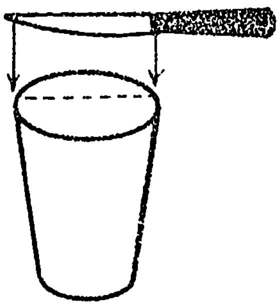
How do each of the plane sections compare with the top and bottom bases of the cylinder it was cut from?

Name or sketch at least three different shapes that a plane could cut if it were passed through a cylinder.

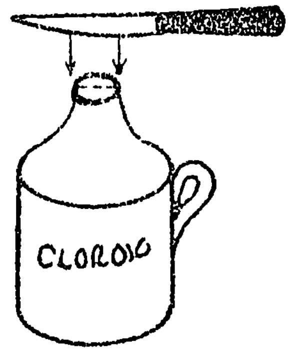
ON YOUR OWN

1. For each of the following sketch the shape of the cut that will result. Draw your sketch beside the figure.

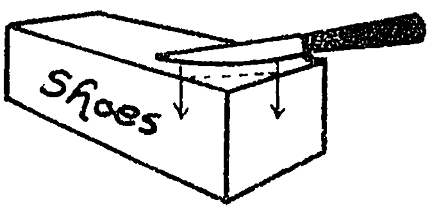
a.



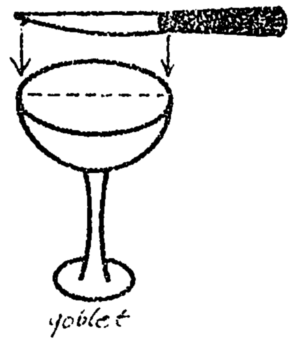
b.



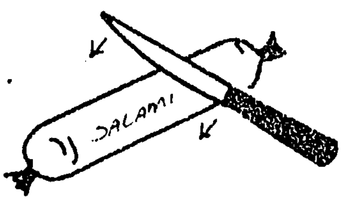
c.



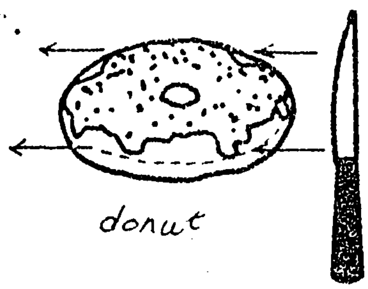
d.



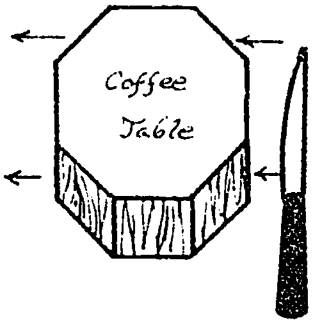
e.



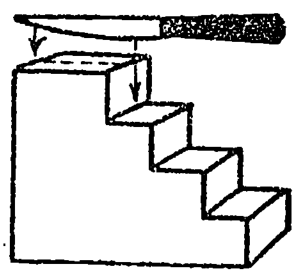
f.



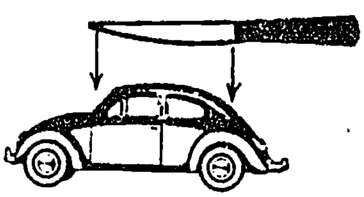
g.



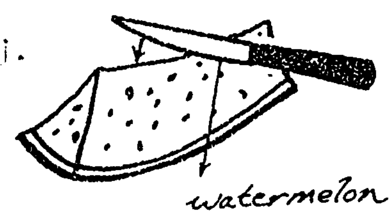
h.



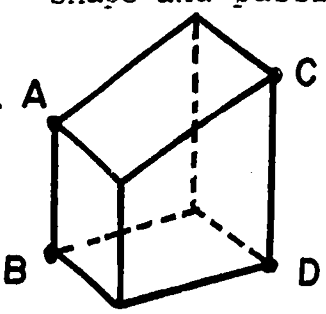
i.

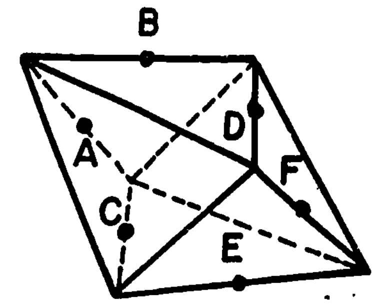


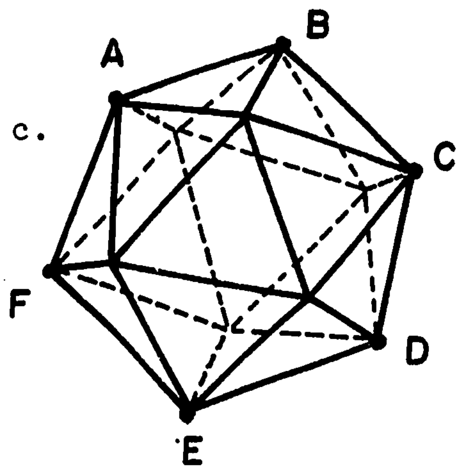
j.

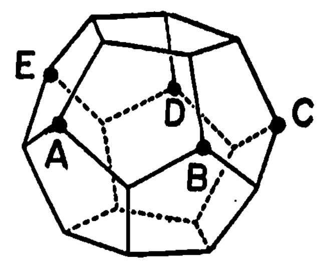


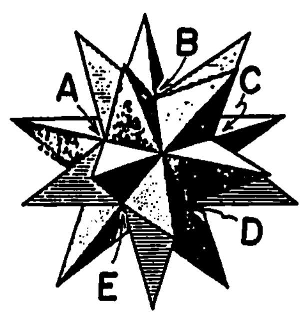
2. For each of the following sketch the shape that would be made by a plane (or a knife) cutting through the shape and passing through the points which are labeled.

a. 

b. 

c. 

d. 

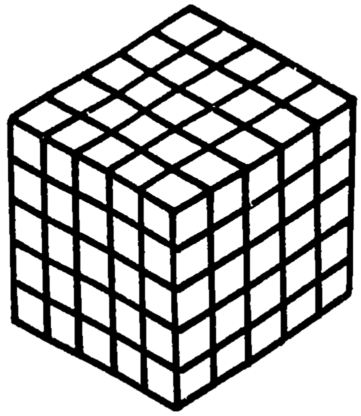
e. 

3. Draw a picture to show how a plane would have to pass through a cube to cut out a triangular shape.

4. Draw a picture to show how a plane would have to pass through a cone to cut an ellipse.

POINT

1. The following $5 \times 5 \times 5$ cube is made from sugar cubes:



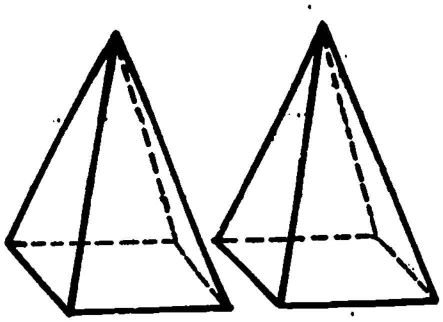
How many sugar cubes were used to build the large cube? ____

If the large cube is painted red and then taken apart, how many sugar cubes will have:

four red faces? _____ three red faces? _____
two red faces? _____ no red faces? _____

2. Every time a plane passes through a sphere it cuts a _______.

3. The picture shows two square pyramids which are the same size and shape. Imagine one plane passing through both of them at the same time. The plane will cut through parallel to the bases of the pyramids. Which of the following statements are true? _______________



a. The two cuts made will be different in shape.
b. The two cuts will both be triangular in shape.
c. The two cuts will be the same shape and size.
d. The two cuts will have the same shape as the bottom faces of the two pyramids.

TAKING A NUMBER BREAK

1. Which of the following numbers are divisible by 9? (Divisible means the number can be divided by 9 with no remainder.) Circle the dots under those numbers you select.

10,062 5,454
111
437
36,036 5,067 17017

35 9,999 7,011
106 1000
640,881 1,008

62 908 80

2. For those numbers which are divisible by 9, find the sums of their digits. (Ex. 5,067→5 + 0 + 6 + 7 = 18). Which of the following is true for all the sums you found? ________

   a. They are even c. They are multiples of 9
   b. They are odd d. They are multiples of 7

3. Find the sums of the digits for at least two of the numbers which are not divisible by 9. Do these sums have the property you selected in question 2?

4. Complete the following: "A number is divisible by 9 if the sum of its digits is ______________________________ ____________________."

5. Which of the following are divisible by 9? ____________

a. 1,278	d. 557,846	g. 312,020,813
b. 13,770	e. 7,080,615	h. 672,537,888
c. 93,907	f. 43,581,274	i. 5,708,915,127

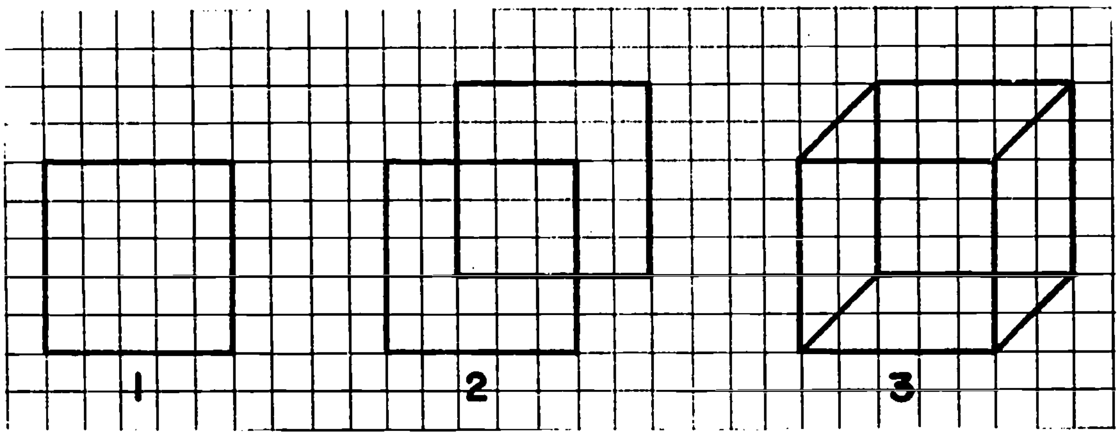
6. Connect the dots you have circled with line segments. Start at the largest number. Connect the numbers in decreasing order. Then return to the starting point. What Platonic Solid would appear if you added four segments to the figure? __________ Can you tell which face of the object is closest to you? Look again.

DEPTH IN 2D.

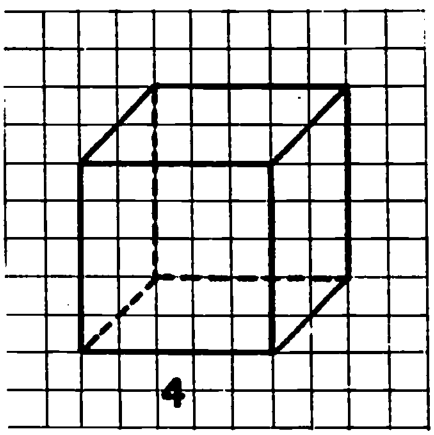
As you have probably found out, it is not easy to draw three-dimensional figures on a sheet of paper (which has only two dimensions). Artists, architects, draftsmen and engineers are often faced with the problem of drawing a three-dimensional scene, building or object on canvas or a sheet of drawing paper. The two-dimensional drawings must give the illusion of three dimensions. There are a number of techniques used in this type of drawing.

Here are some tricks which can be used to draw simple space figures. Study them.

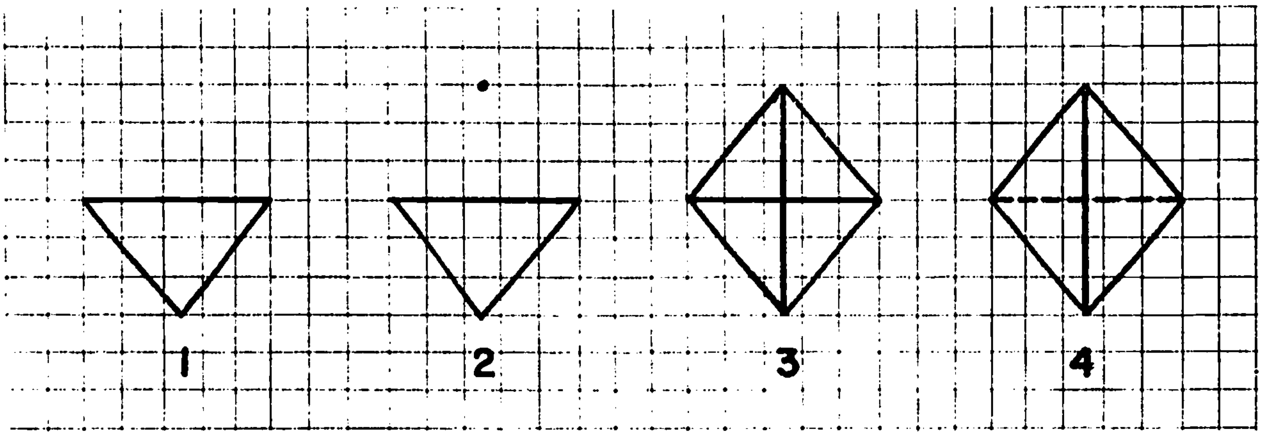
Cube



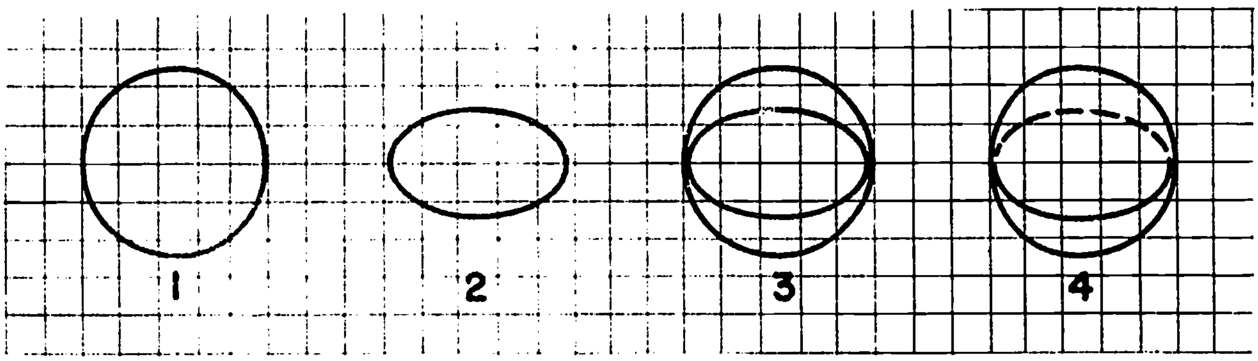
If the cube is solid the back edges would not be visible to the human eye. The lines which are not visible in this view are dotted.



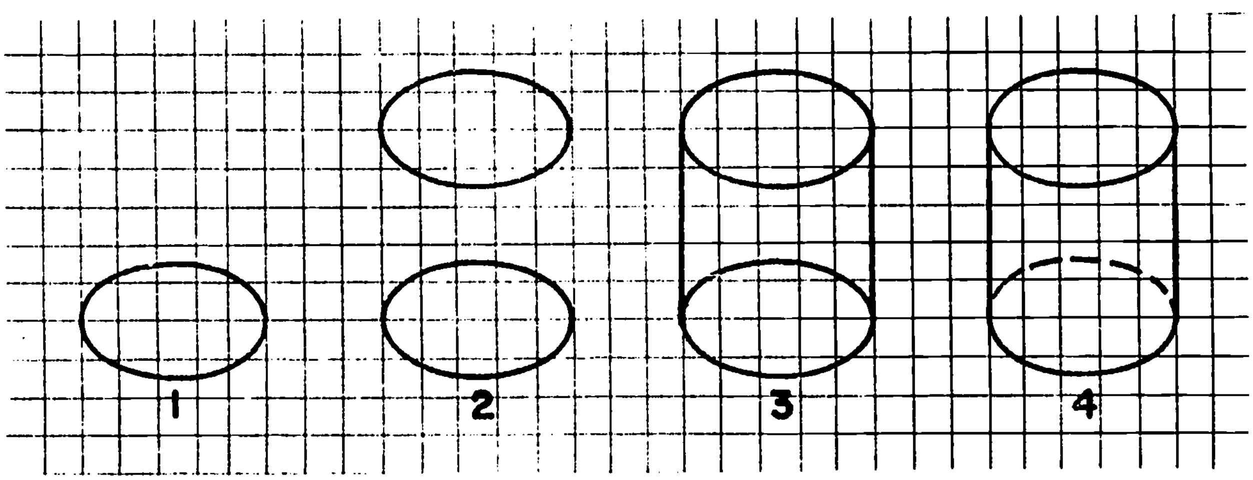
Triangular Pyramid



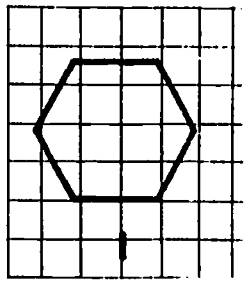
Sphere

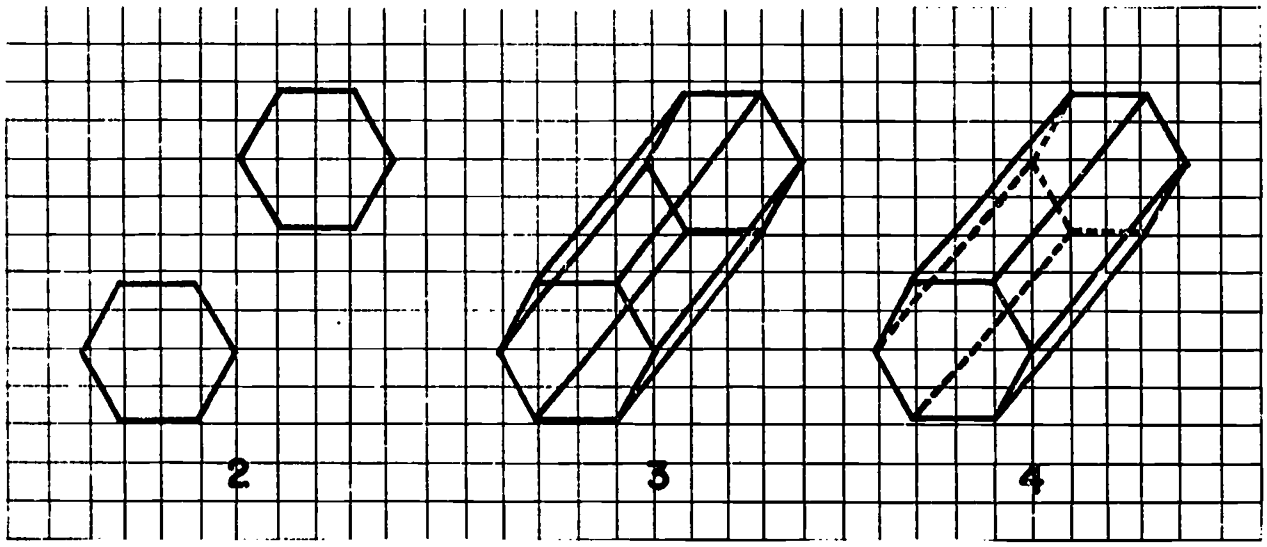


Cylinder

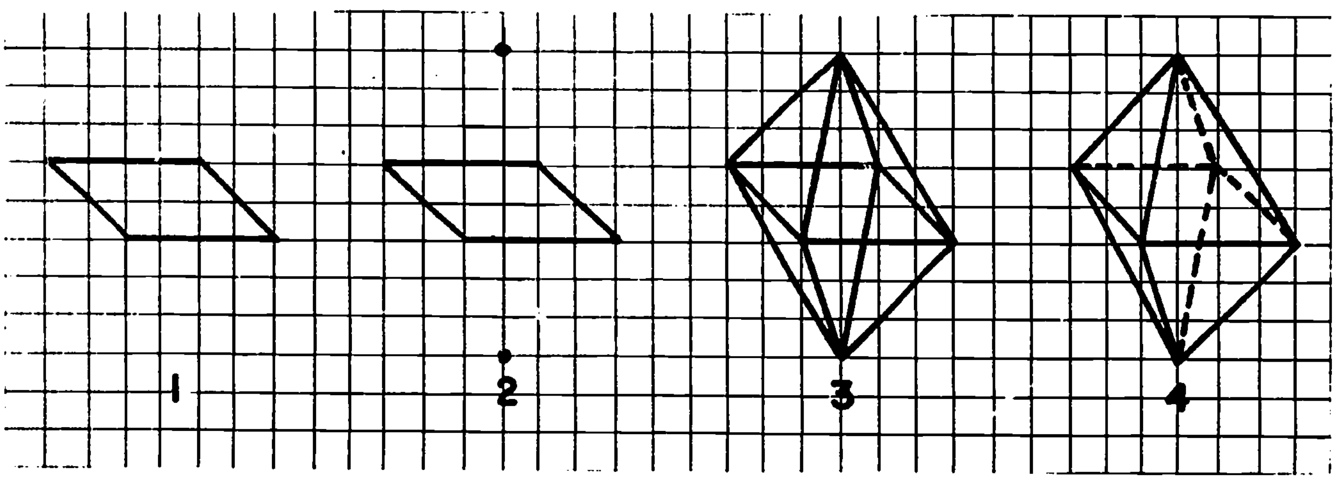


Hexagonal
Prism





Octahedron



ON YOUR OWN

The inserts at the back of the book are for your drawings. They are sheets of engineer's drawing paper. Draw your figures on the blank side; use the tiny squares on the other side as guidelines. You will need a straightedge.

1. Use the trick techniques illustrated to draw:

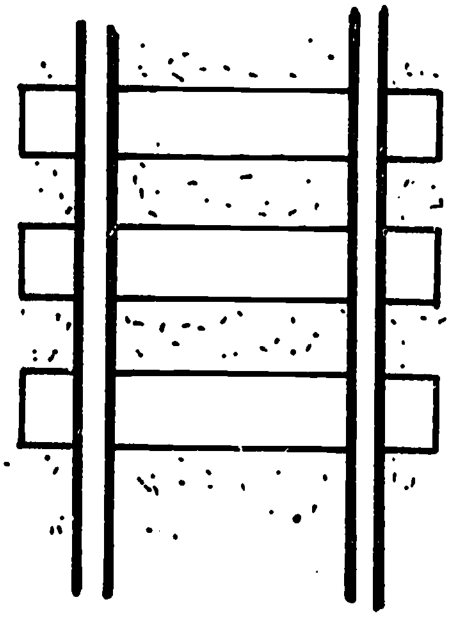
   a. two cubes of different size
   b. a triangular pyramid
   c. a sphere
   d. a cylinder
   e. a cone
   f. a triangular prism
   g. an octahedron
   h. a shoe box

2. In number 1 you were asked to draw single objects. Use the same techniques to draw:

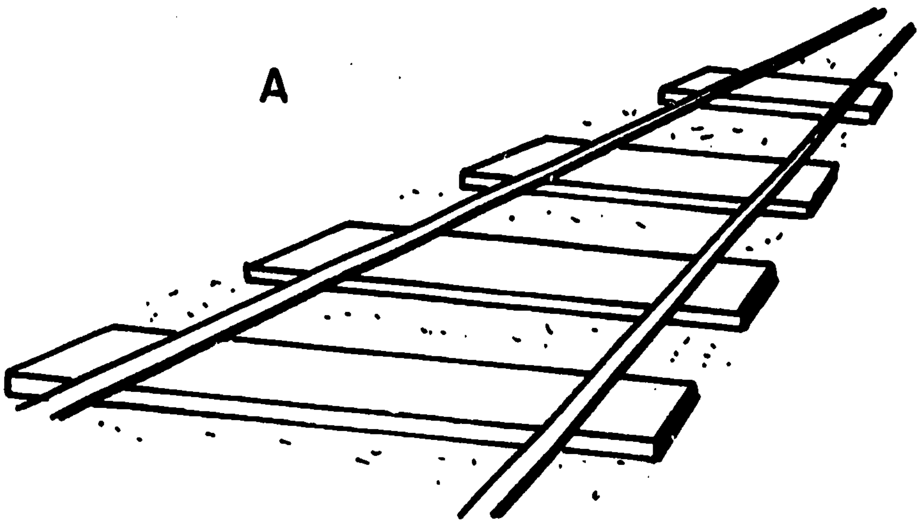
   a. two cubes (same size) which share an edge
   b. a double-dip ice cream cone
   c. a cube with a circular tunnel drilled through the center
   d. two tetrahedrons which share a face
   e. the Geopolis Micro-Film Library (see page 2)

3. (Optional) Draw a collage which includes at least five different three-dimensional objects.

PUTTING SOLIDS IN PERSPECTIVE



A



B

A class was asked to draw a picture of railroad tracks. The pictures above were done by two different students.

THINK ABOUT IT

1. Railroad ties are actually the same size. Which picture portrays this notion better?

2. The steel rails of railroad tracks are the same distance apart at all points. Which picture portrays this better?

3. Which picture do you think is more correctly drawn?

4. If you were standing on the tracks and looking off into the distance, which picture best portrays what you would see?

5. Where would you have to be to see the railroad ties as they are drawn in Figure A?

The student who drew picture B used a technique called perspective. This technique is used by artists to achieve a feeling of space, of depth and of the third dimension within the limits of a flat drawing surface.

ONE-POINT PERSPECTIVE

Steel rails of a train track are always the same distance apart. If you were to stand on the tracks (we don't suggest you try this) and look off into the distance, the rails would seem to come together at a point.

This illusion is an example of one-point perspective. Parallel lines which extend away from the observer seem to meet at a point (vanishing point) on a line which is at eye-level (horizon line).

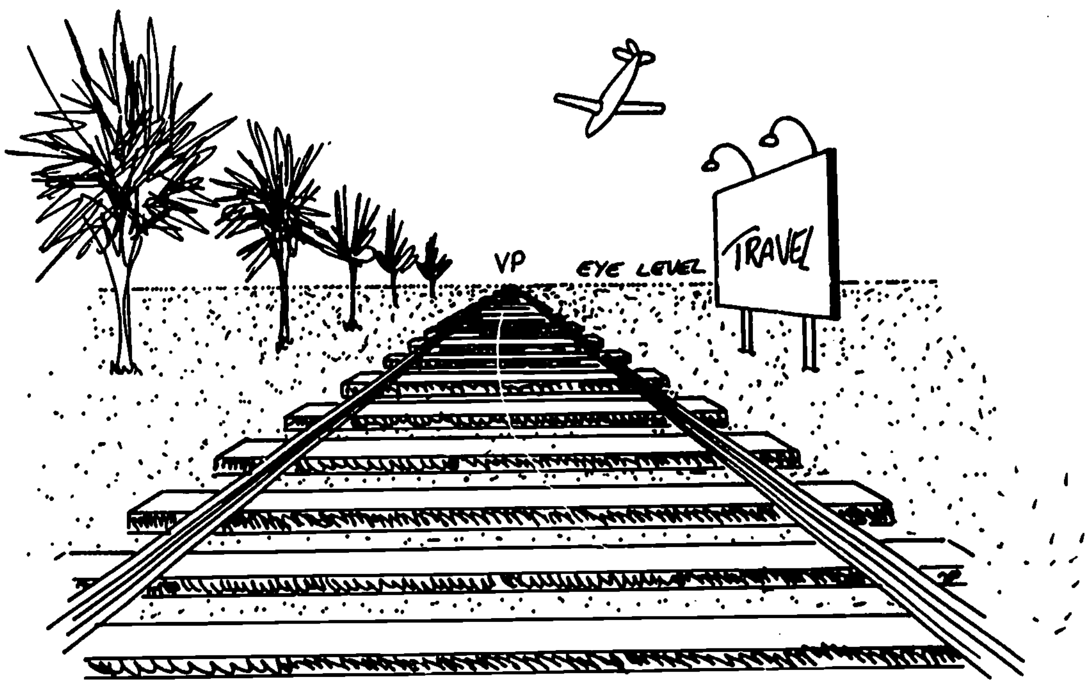


figure 1

This one-point perspective technique is frequently used by interior designers who want to describe their plans for decorating a room.

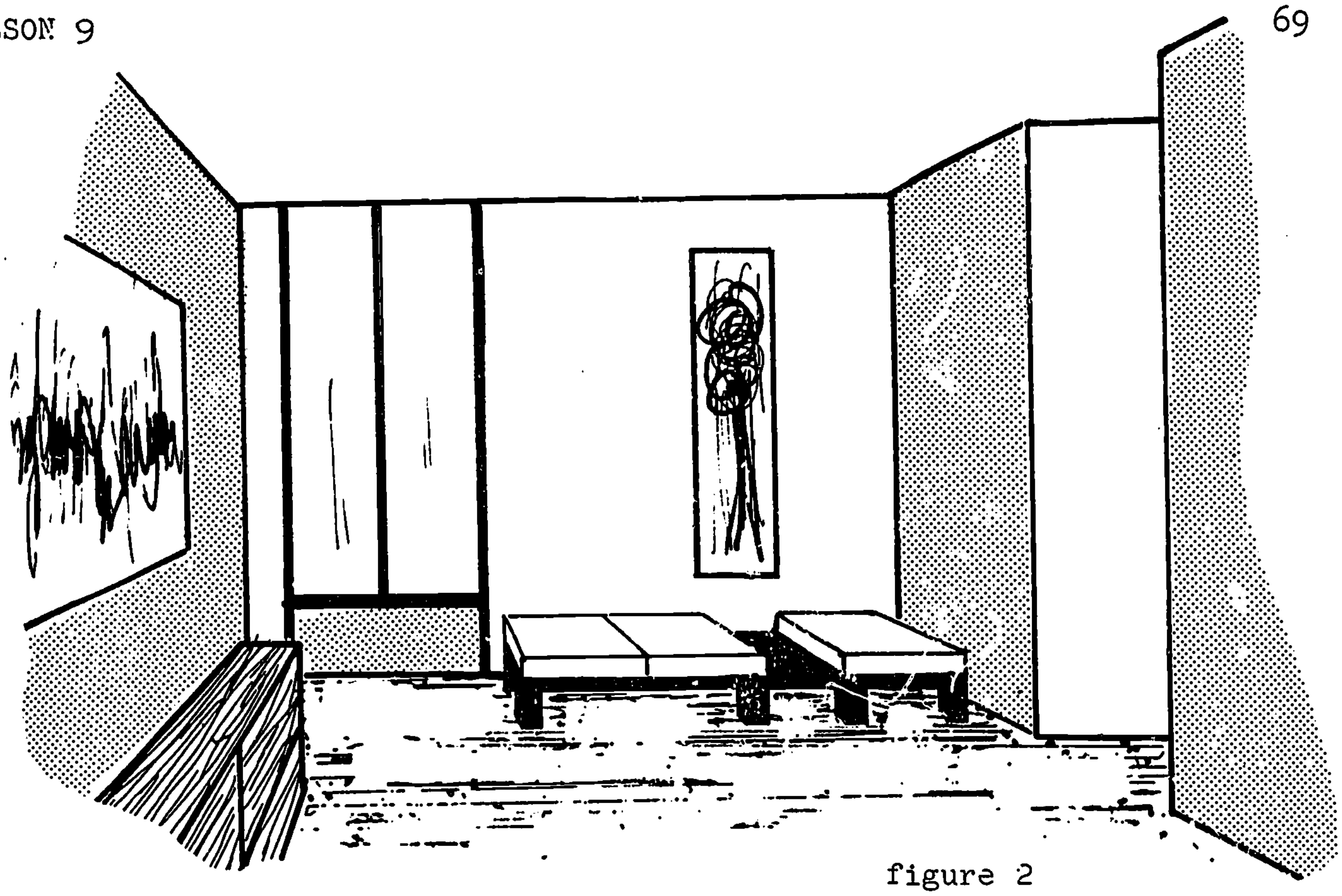


figure 2

Notice how the lines of the various objects (including the room) which in reality are parallel, converge to a point (vanishing point). This point is located along the eye-level line.

THINK ABOUT IT

1. In figure 1 where is the vanishing point located with respect to the center of the picture? Where is the viewer standing?
2. Are you looking at the railroad tracks from the top or the bottom? Where are the tracks located with respect to the horizon line (above or below)?
3. Do you see the top or the bottom of the airplane? Where is the airplane located with respect to the horizon line?
4. In figure 2, is the vanishing point at the center of the picture? Where is the viewer standing?

Something To Keep In Mind:

The selection of a vanishing point on the horizon line will depend on the position of the viewer.

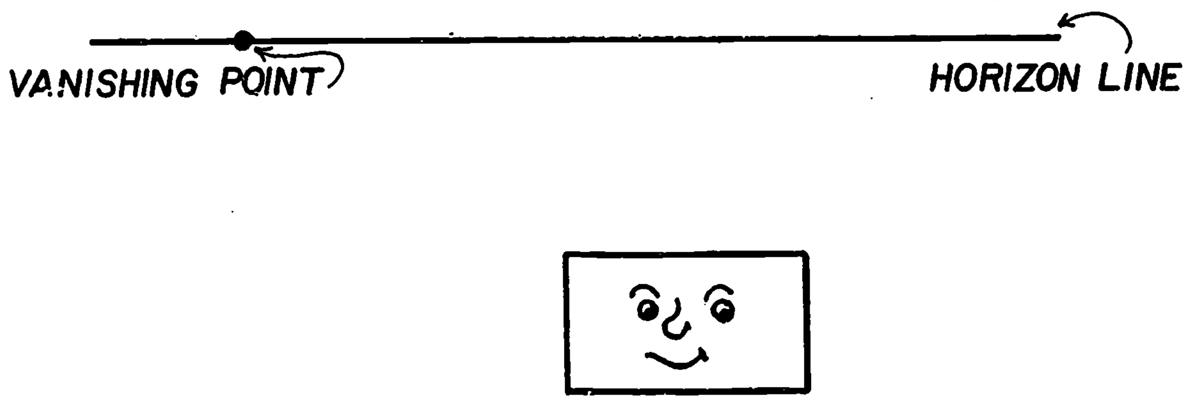
Placing an object above or below the horizon line will depend on whether the viewer is seeing the top or bottom of the object.

Study the following steps for drawing a box using the one-point perspective technique.

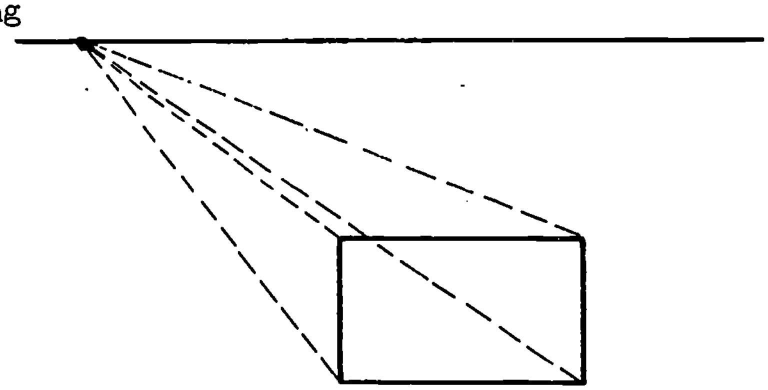
1. Draw your horizon line.

2. Select a vanishing point.

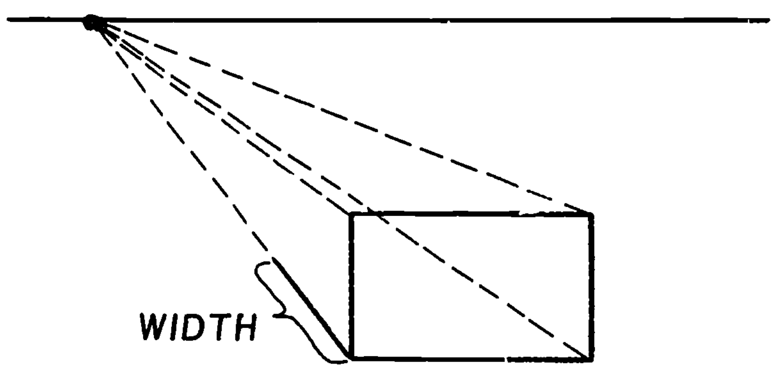
3. Draw the front face of the box.



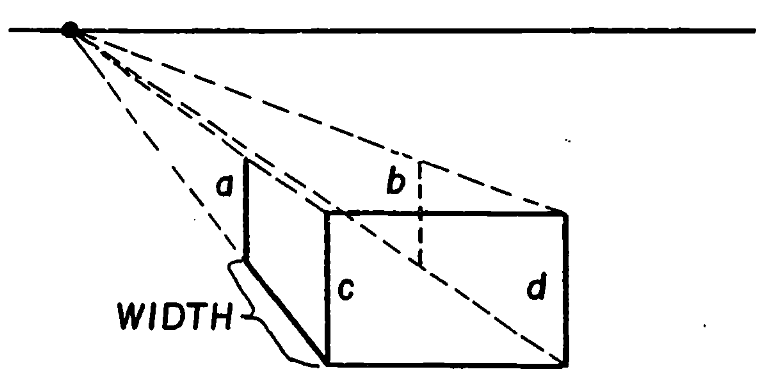
4. From each corner of the front face draw a dotted line to the vanishing point.



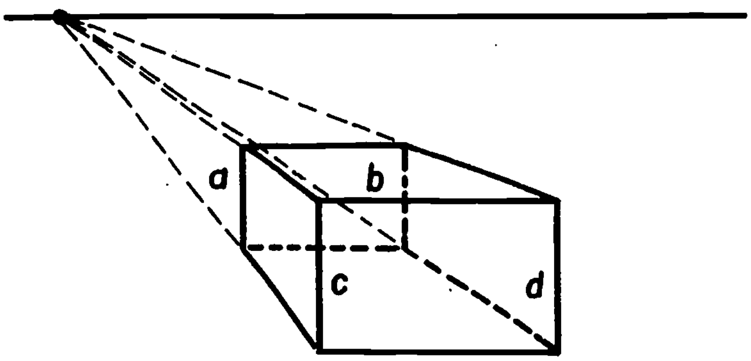
5. Select a width which is pleasing to your eye.



6. Draw in edges a and b parallel to front edges c and d.



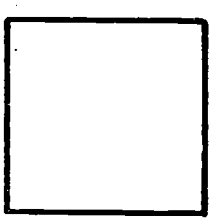
7. Complete the box by drawing in the other edges. Remember — those lines which are hidden should be dotted.



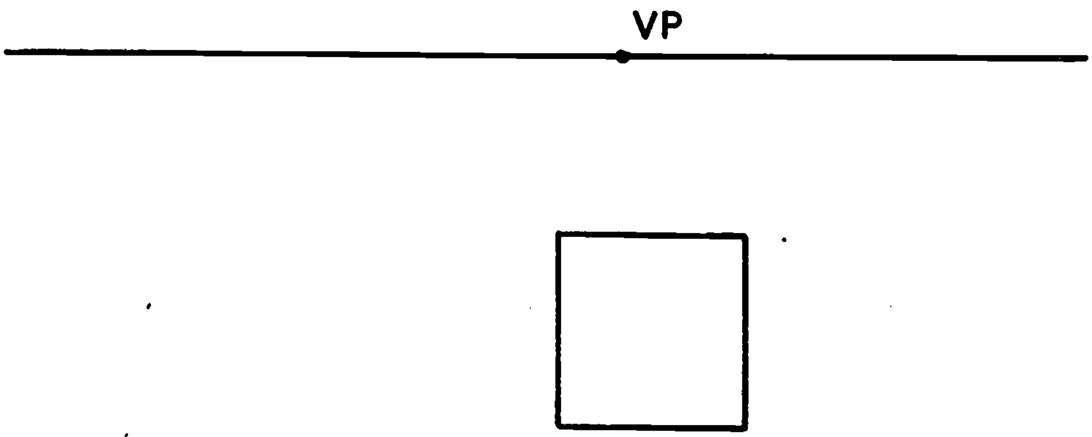
ON YOUR OWN

1. Use the one-point perspective technique to draw three different views of a cube.

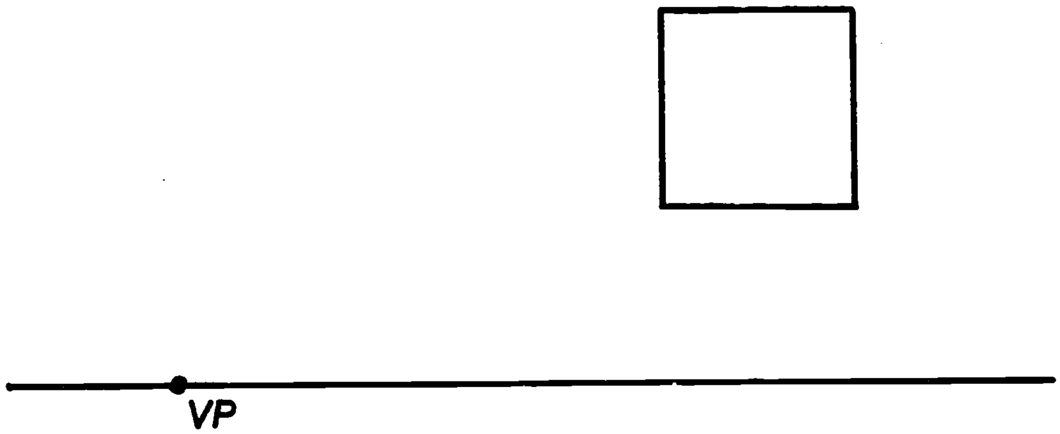
A.



B.



C.



(For exercises 2, 3, 4 and 5 use the engineer's paper which is included at the back of this booklet.)

2. Draw a picture of a pair of dice using the one-point technique.

3. Draw a picture which gives the illusion of looking down a hallway or tunnel. Locate the vanishing point at the center of the picture.

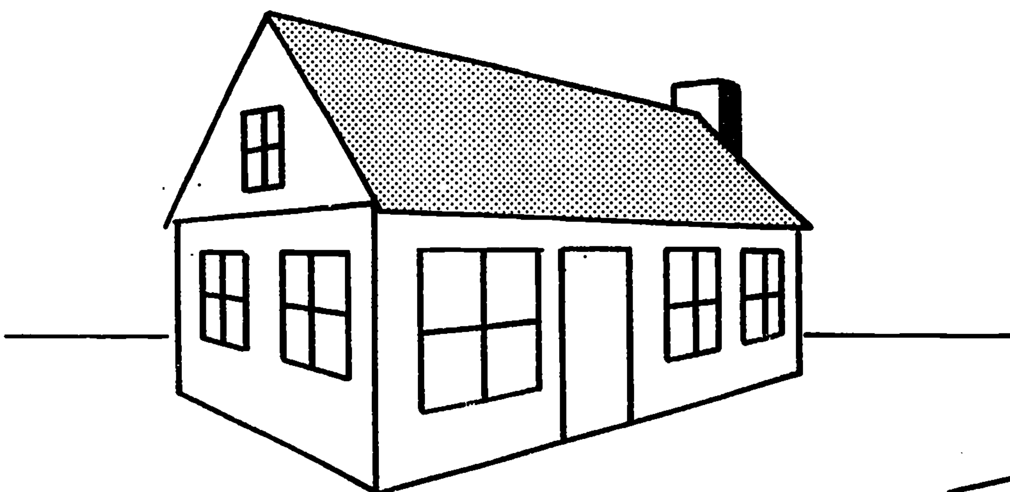
4. Sketch a picture of your school or some other building using the one-point technique. Place the vanishing point off center — to the right.

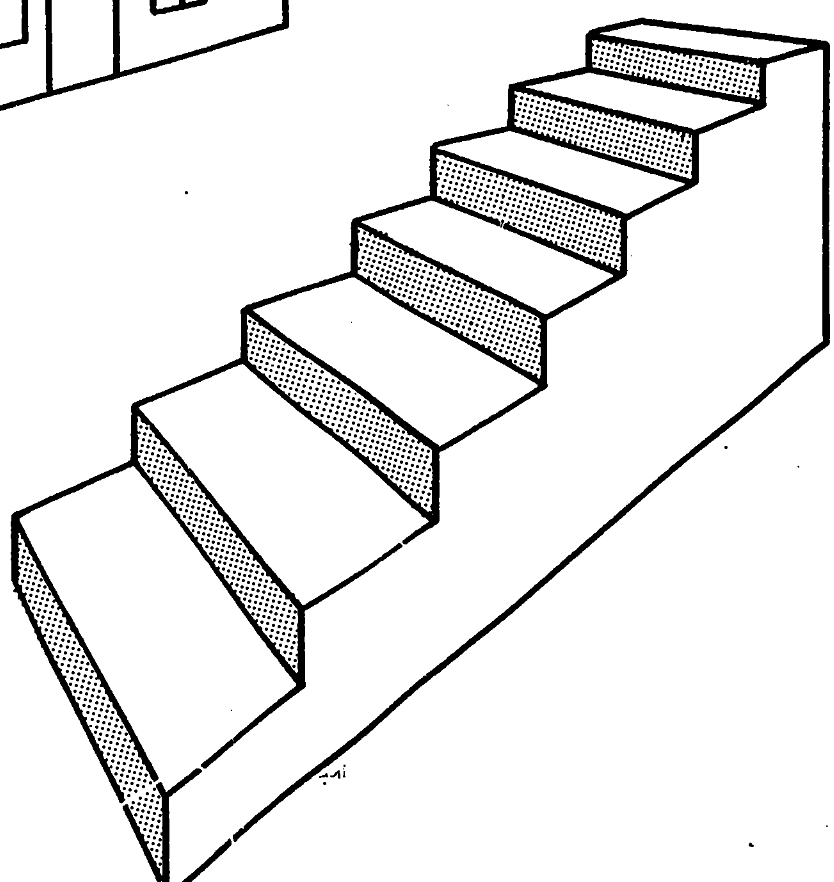
5. Sketch a picture of a highway, country road or a busy city street using one-point perspective.

TWO-POINT PERSPECTIVE

Artists and architects more frequently use a two-point perspective technique in their drawings of three-dimensional objects. In this form of perspective drawing, the parallel lines of an object converge to two different points along the horizon (eye-level line).

The following are examples of the use of the two-point perspective technique.

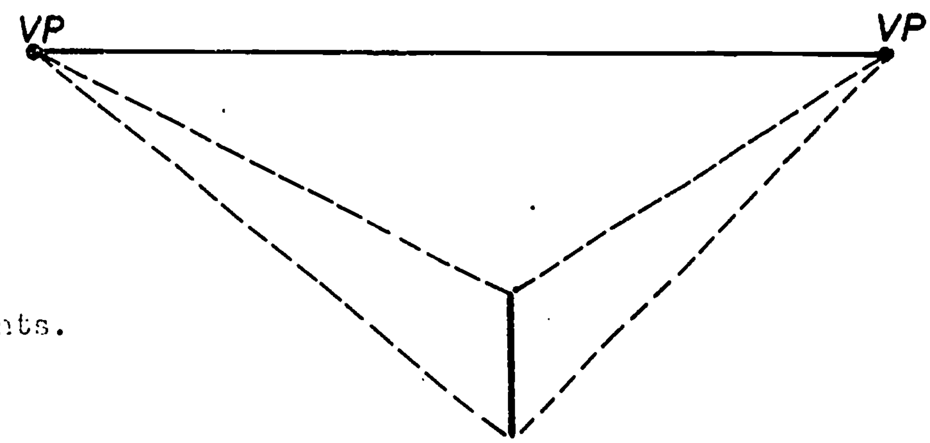




Study the following steps for drawing a box using two-point perspective.

1. Draw your horizon line.

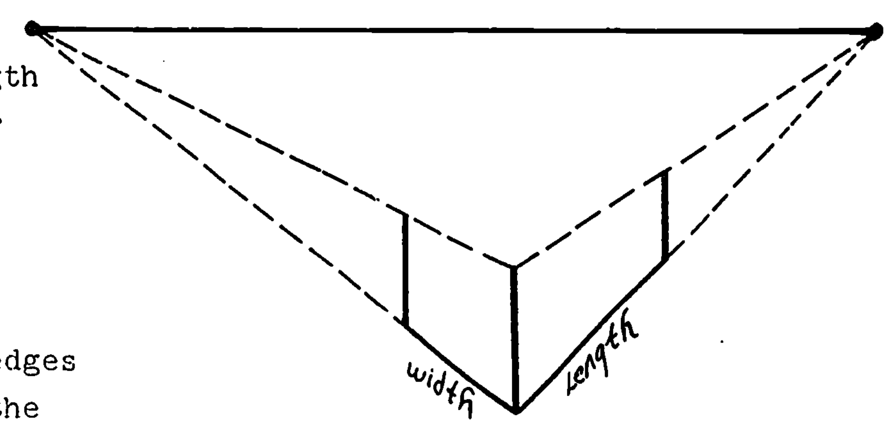
2. Select two vanishing points.



3. Draw an edge of the box.

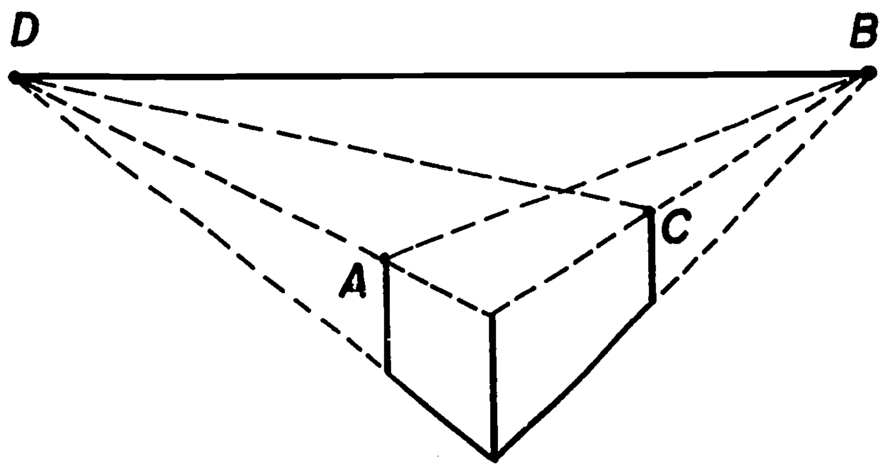
4. Connect the endpoints of the edge to the vanishing points.

5. Select a length and width for the box.

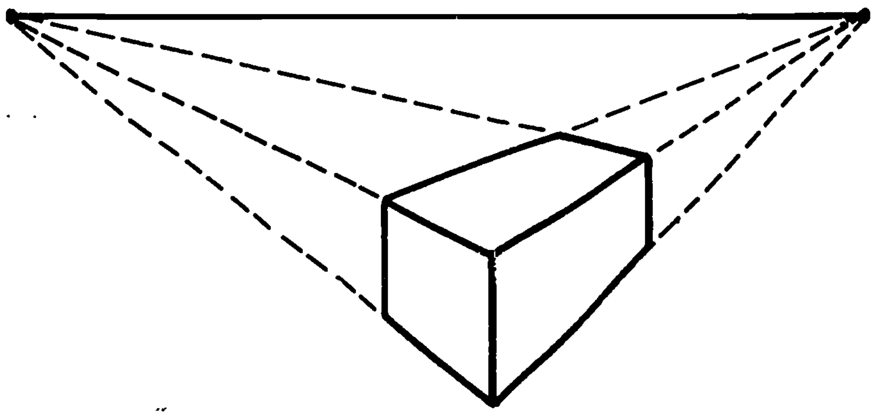


6. Draw in the edges parallel to the front edge.

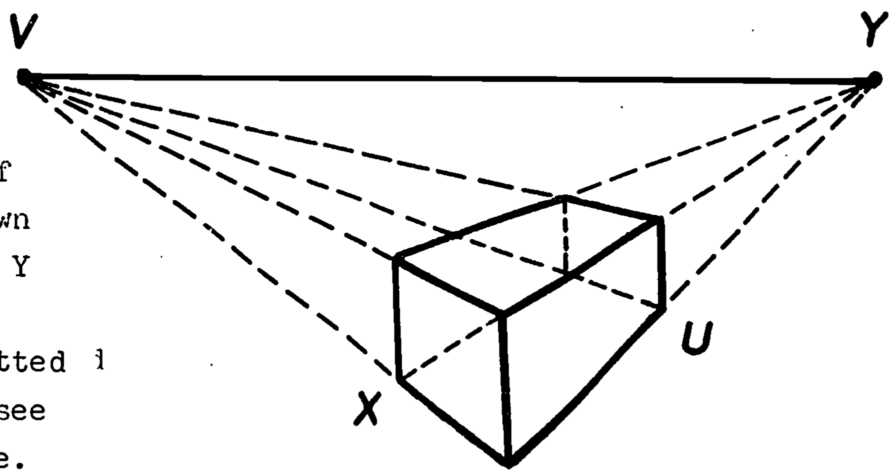
7. Draw a dotted line from A to B and from C to D.



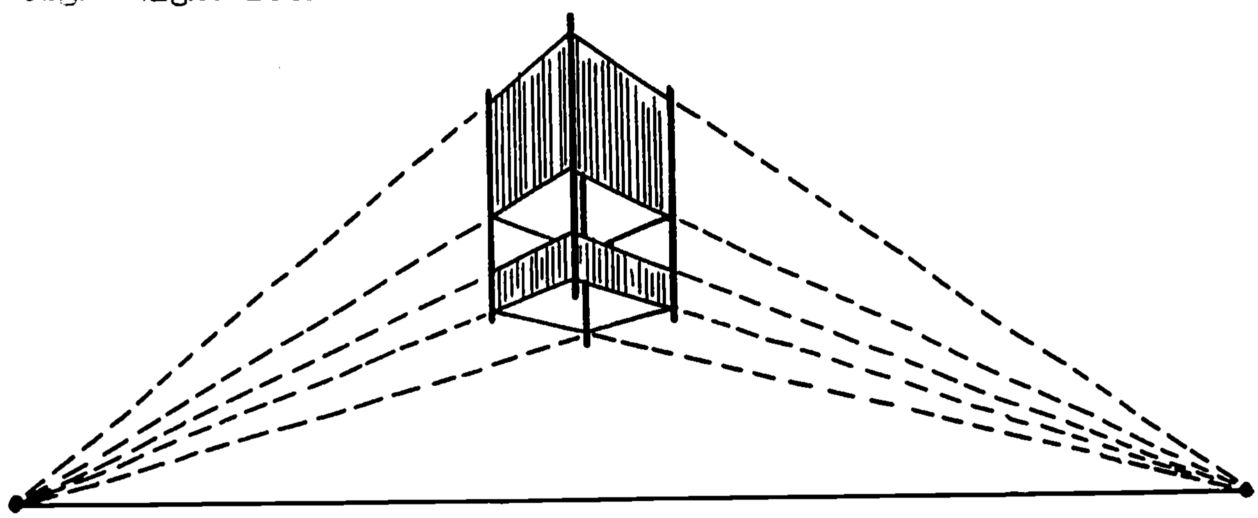
8. Draw in the edges of the box which you can see.



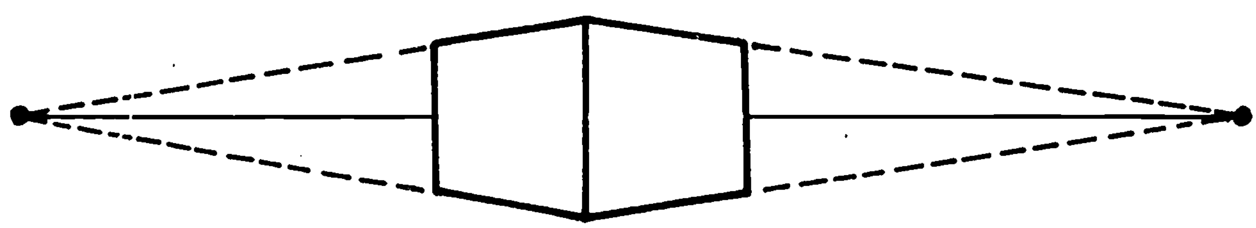
9. The hidden edges of the box can be shown by connecting X to Y and U to V. These edges should be dotted for you could not see them from this side.



A picture of a box kite in the air above the line of sight might look like:



If you are drawing a picture of a box which hides part of the horizon it might look like:

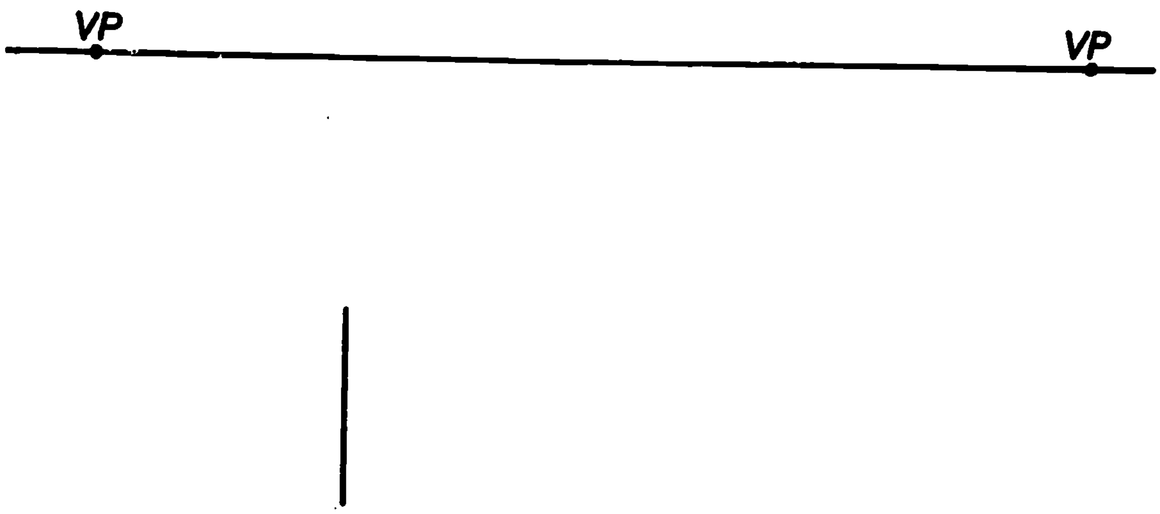


Again...

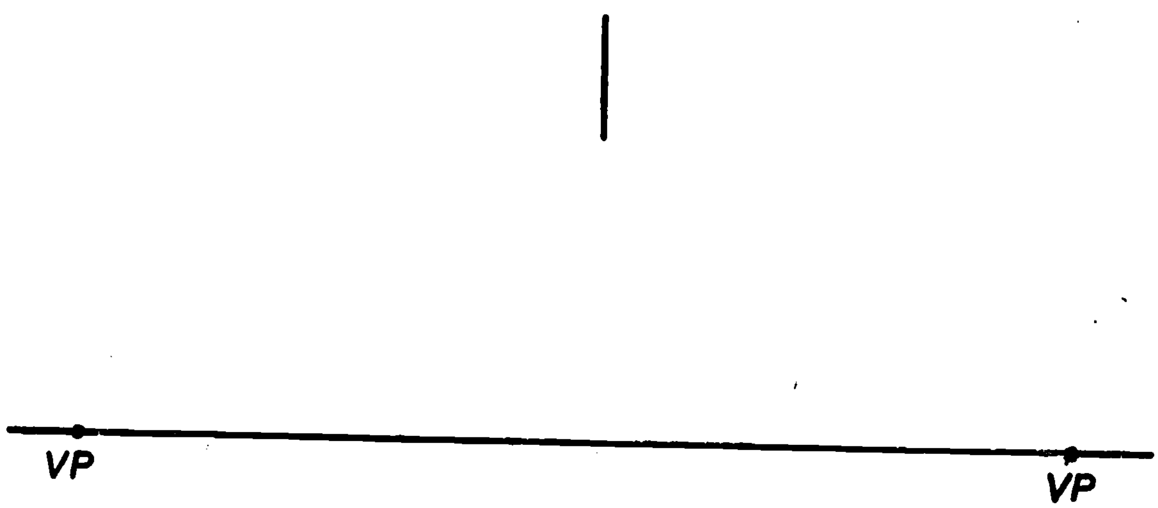
1. The location of the vanishing points depends on the position of the viewer.
2. The placement of an object above, below or at the horizon line depends upon whether you want to show the top, bottom or just the sides of an object.

ON YOUR OWN

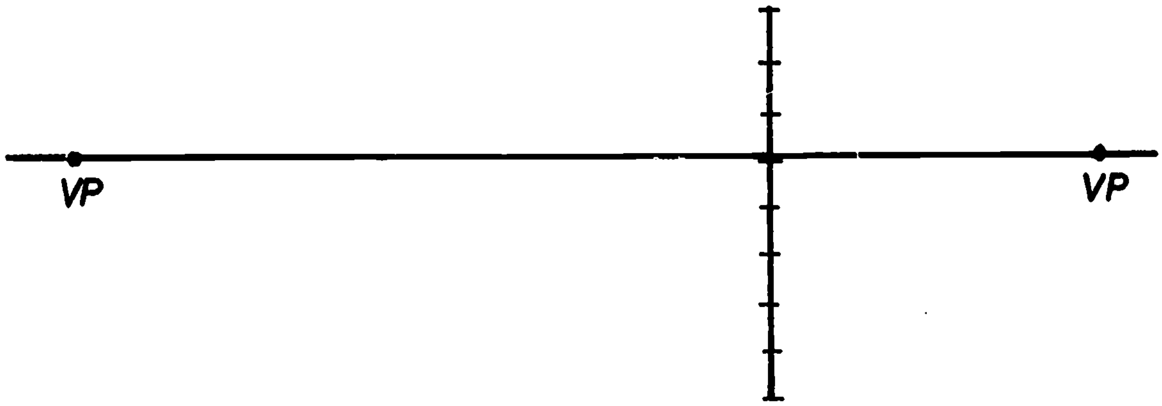
1. Use the two-point perspective technique to draw a picture of a cube.



2. Use the following to draw a picture of a shoe box.



3. Use the following to draw a picture of a eight story rectangular-shaped building.



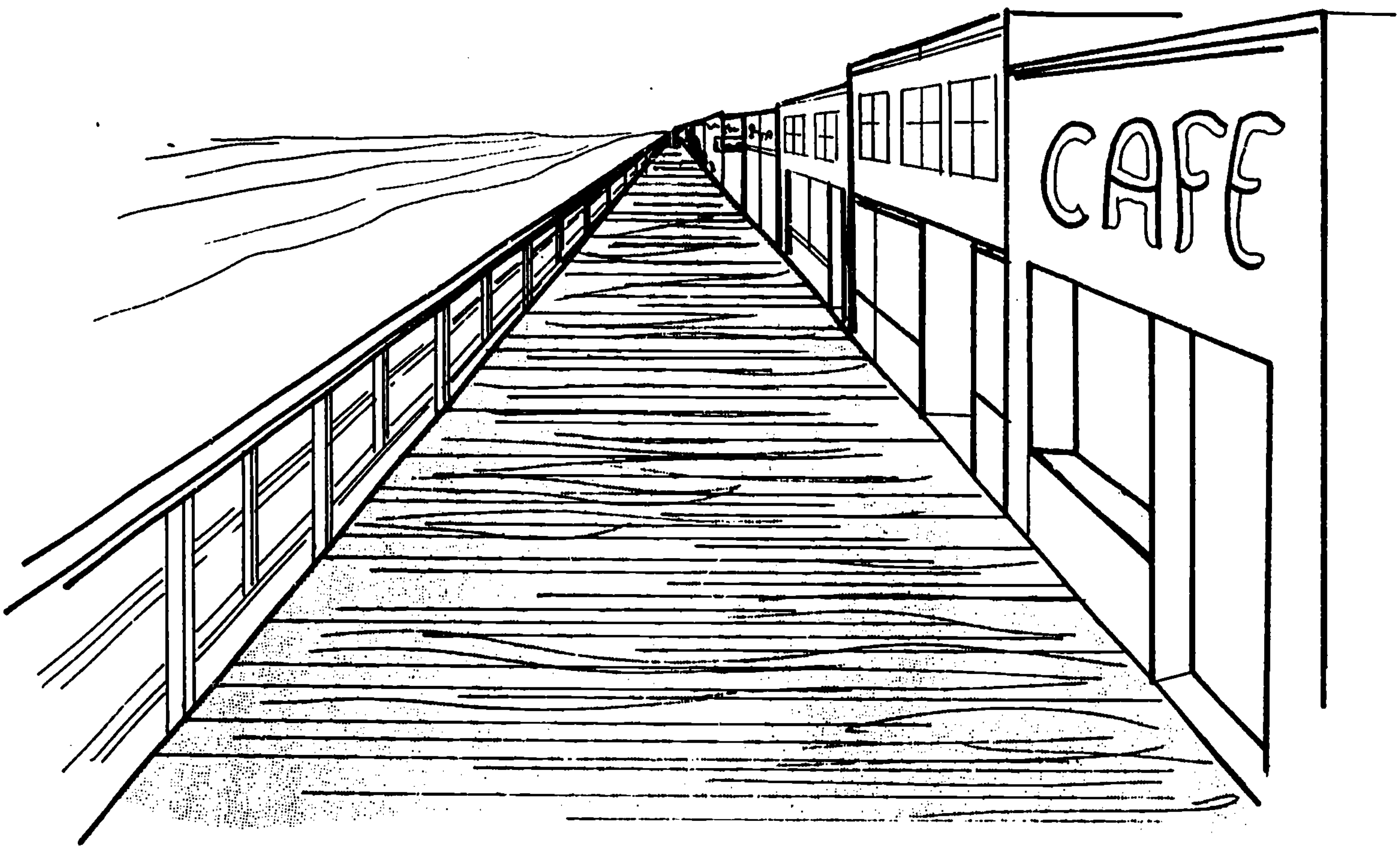
4. Draw pictures of the seven Soma Cube puzzle pieces that you built in Lesson 6.

5. Draw a picture of a staircase using two-point perspective.

DISTANCE ILLUSION

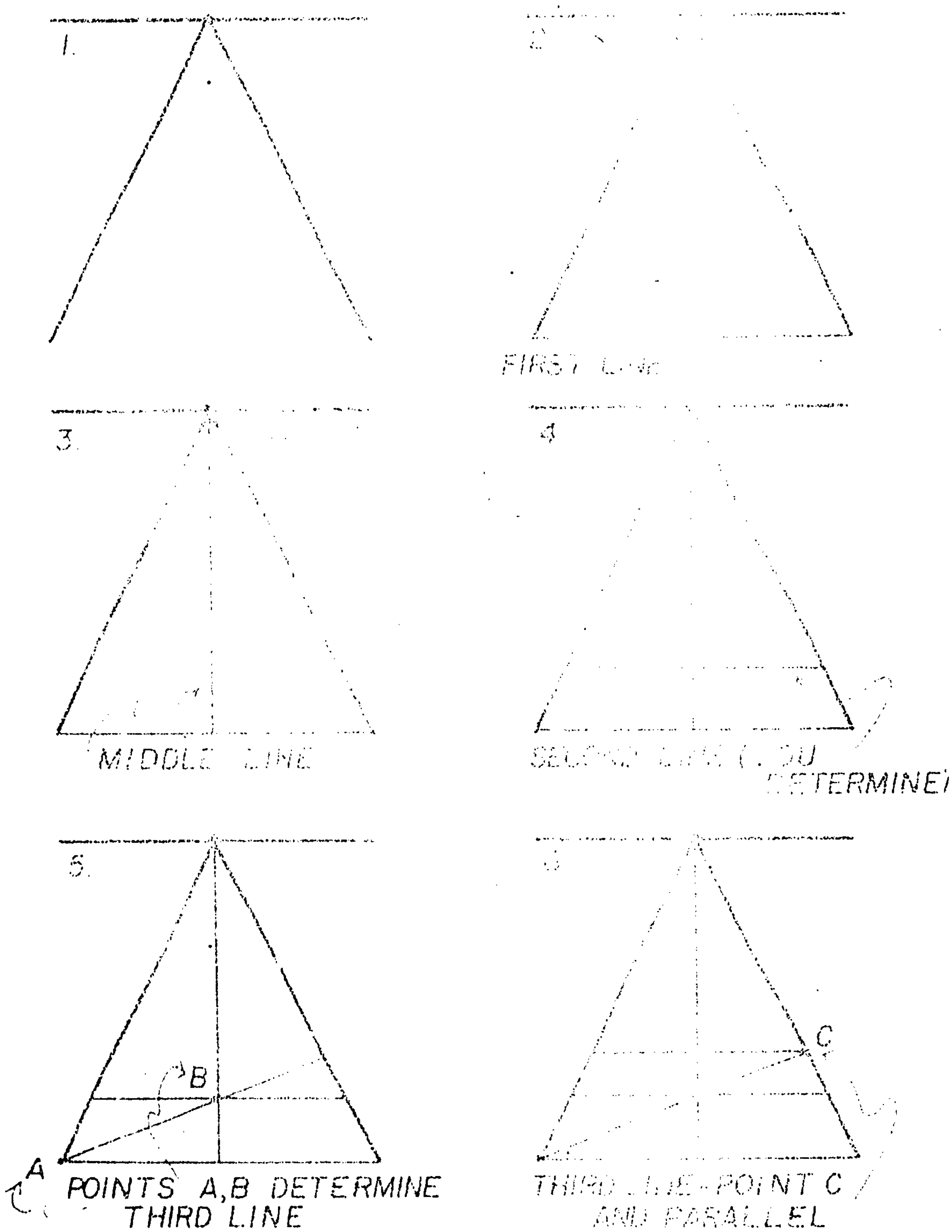
Another aspect of perspective drawing is to picture objects so they appear to be at different distances from the viewer. This is accomplished by making objects smaller as they recede from the viewer.

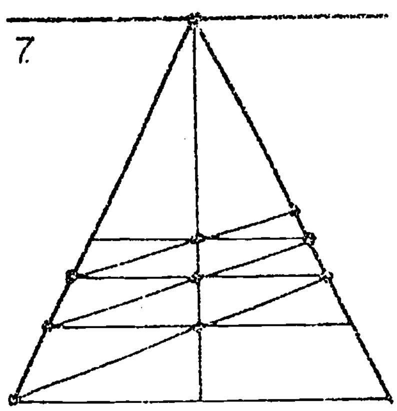
For example, an artist sitting at the entrance to the boardwalk in Atlantic City might sketch the scene like this:

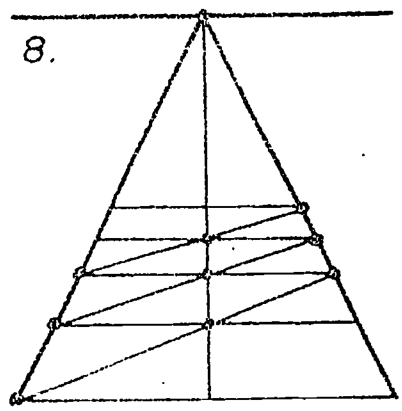


The artist knows that the wooden planks are all the same size to give the illusion of distance. The same is true of the side rail posts.

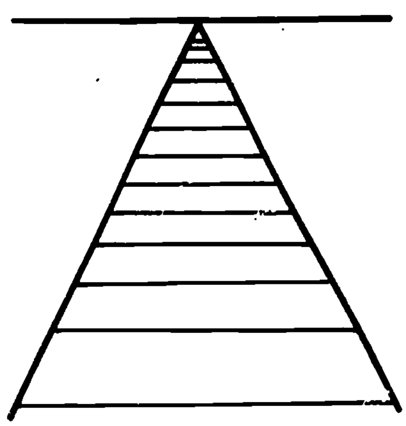
The following steps show a method which can be used to achieve this distance effect with objects which are parallel to the horizon line. The trick to using this technique is spacing lines in relation to the vanishing point.



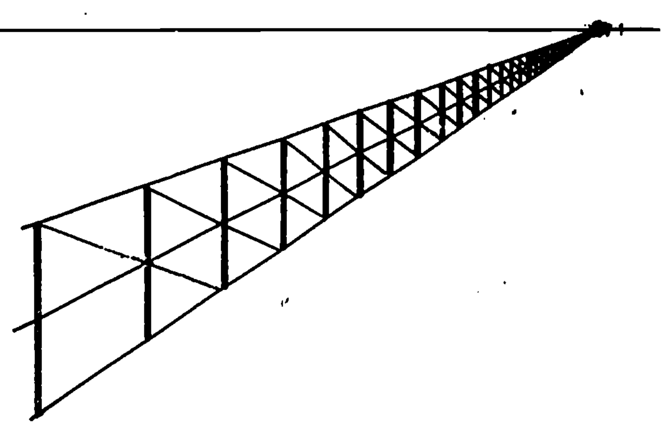




If you continue the process and erase the middle line and the diagonal lines you will get a picture which appears to be train tracks or the planks on a boat dock. (Hint-- do you need to keep drawing the diagonal lines?)

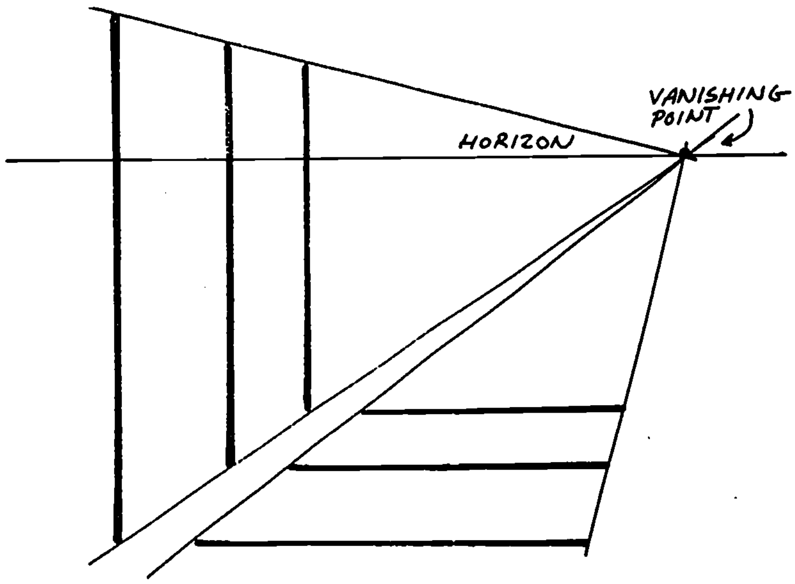


This same process can be used with objects which are situated perpendicular to the horizon line (side rail posts, telephone poles).

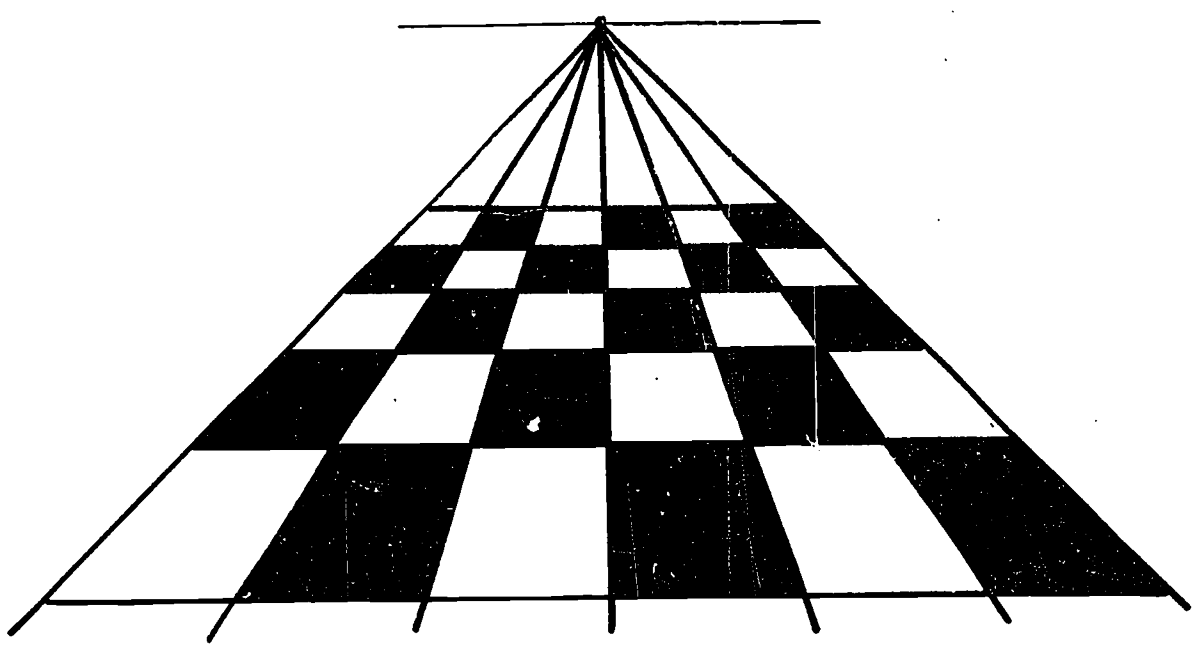


ON YOUR OWN

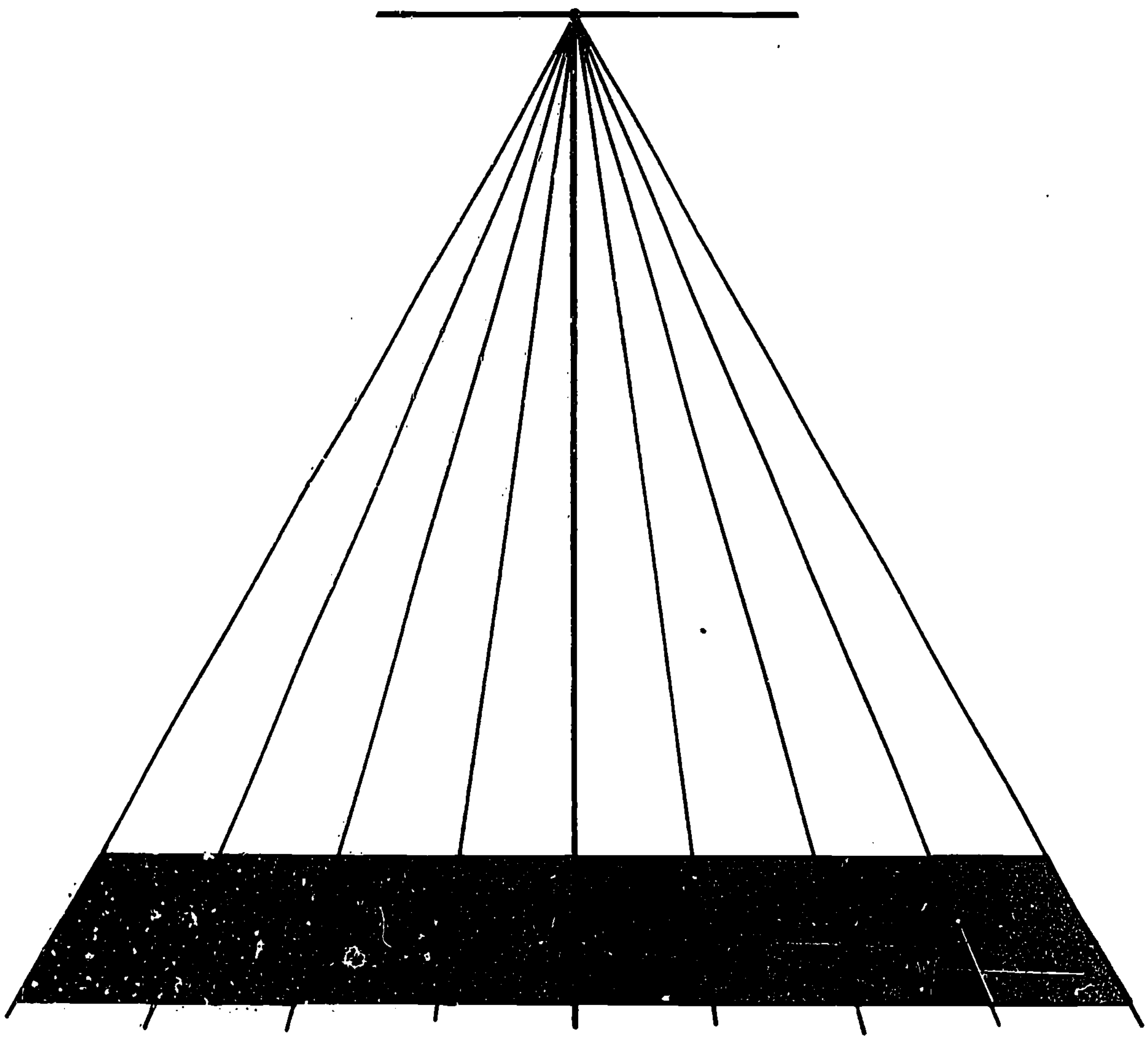
1. Use the perspective technique just described to complete this picture of train tracks with telephone poles along one side of the tracks.



2. A checkerboard, chessboard or tile floor may also be drawn using this perspective technique.



Complete the picture of an 8×8 checkerboard below: (Hint--use the technique described to place your horizontal lines.)



3. Use the distance illusion technique and either the one-point or two-point technique (or both) to draw the following:

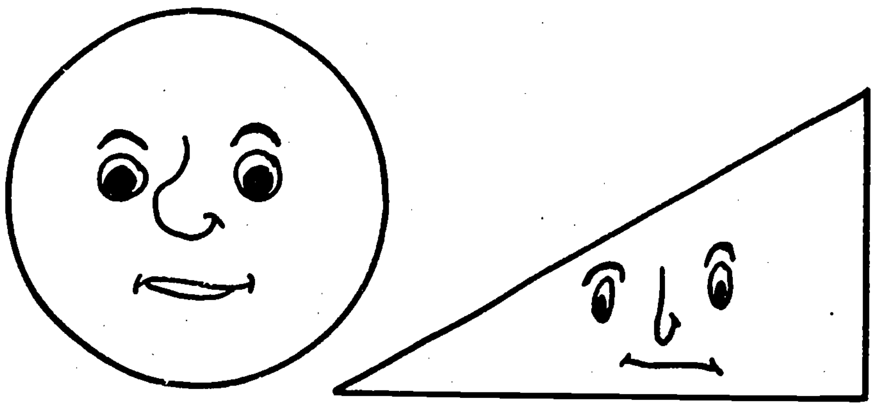
   a. a scene depicting a country road (you might include billboards, telephone poles, picket fences or a line of cars moving down the road away from the viewer).

   b. a scene showing the buildings along a street either in a downtown area or in a residential area.

   c. a scene showing people waiting in line to get tickets for a movie.

THE $\frac{3D}{-1D}$ WORLD

Suppose the world we live in suddenly lost a dimension. Without warning, the dimension of breadth would be eliminated. Imagine yourself an inhabitant of this two-dimensional world.



Do you often get that flattened out feeling?

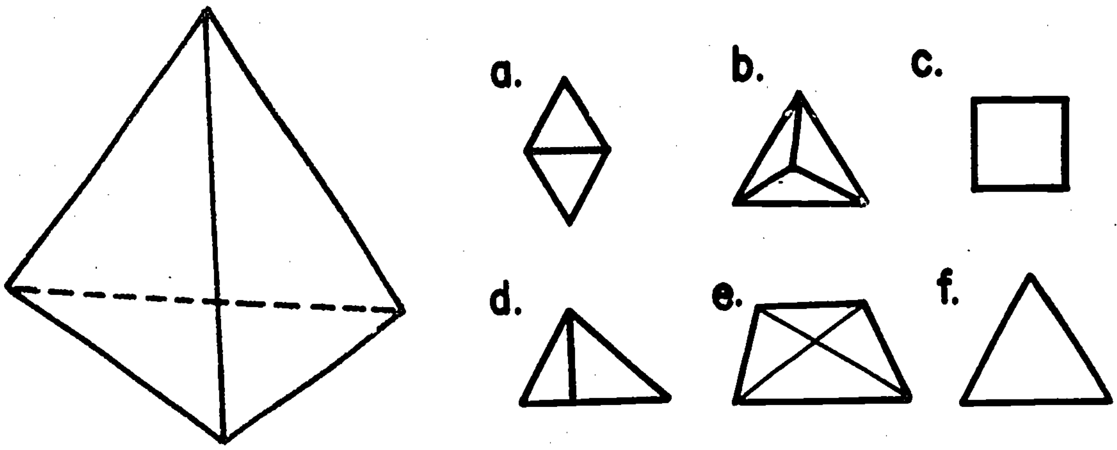
Class Discussion:

1. Describe your new world.
2. What would you look like?
3. How would your life be different in the world of two dimensions?
4. What would the buildings of Geopolis look like in a two-dimensional world?

Let's call our new two-dimensional world Shadowland. Think about standing in front of a lamp and letting the light rays project your image onto the wall (a flat two-dimensional surface). What do you see? If you change your position, what happens? Is your shadow a good picture of the way you would look in a two-dimensional world?

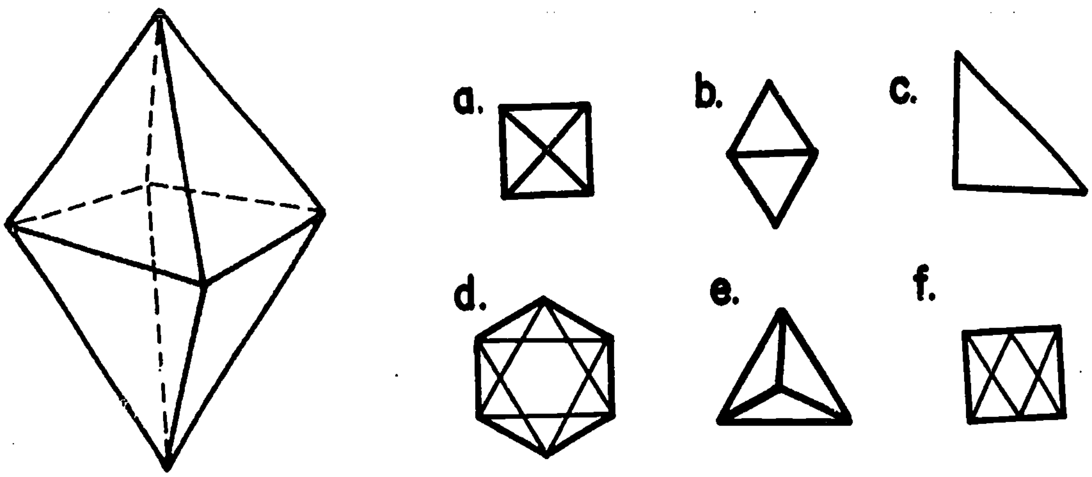
EXERCISES

1. Which of the following shadow shapes do you think can be produced by holding a straw model of a tetrahedron in different positions in front of a light source? ________



TETRAHEDRON

2. Which of the following shadow shapes do you think can be produced by shining a light on a straw model of an octahedron held in different positions? ________



OCTAHEDRON

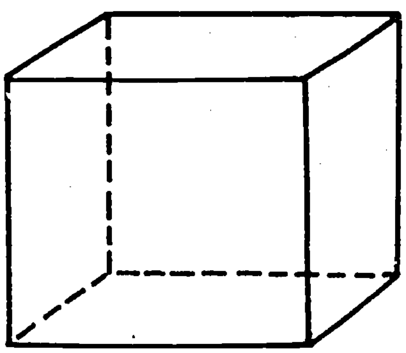
CLASS ACTIVITY

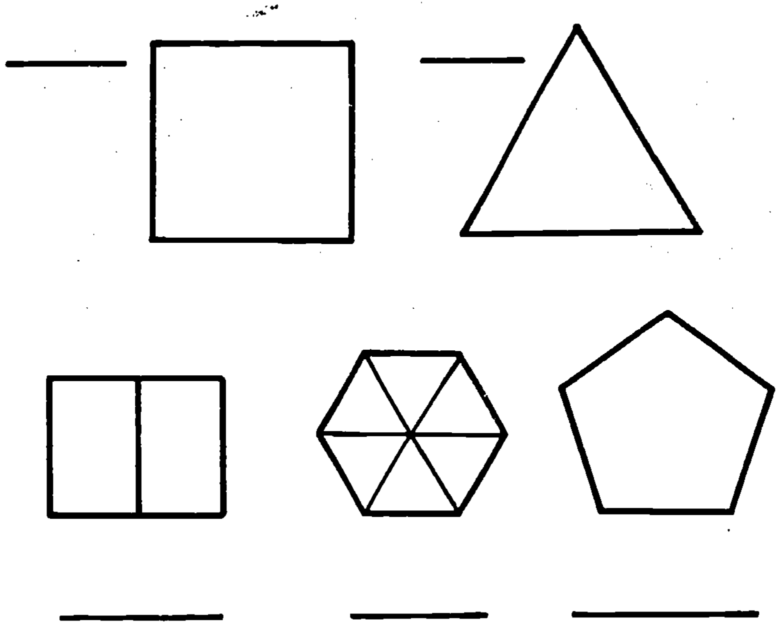
Together with your teacher and your classmates, experiment to see how many different shadow shapes can be produced by shining a light on some of the straw models you constructed in Lesson 1. Use the lamp of the overhead projector as a light source. Below is a checklist which gives a picture of the three-dimensional figure and possible shadows which might be produced by holding the figure in different positions in front of the lamp. Check the shapes which you see on the screen. (First you might check your answers to the preceding exercises by experimenting with the tetrahedron and octahedron.)

Space Figure

Possible Shadows

1.

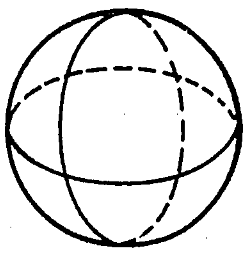




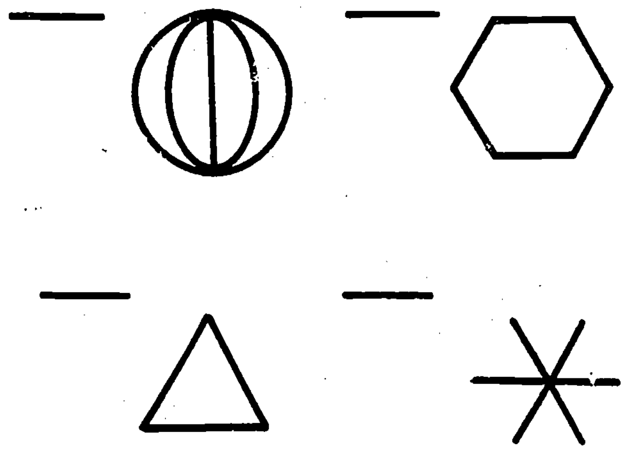
(For each of the space figures, if you see any other shadows which are not illustrated sketch them in any available space.)

Space Figure

2.

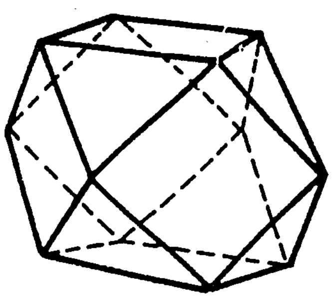


Possible Shadows

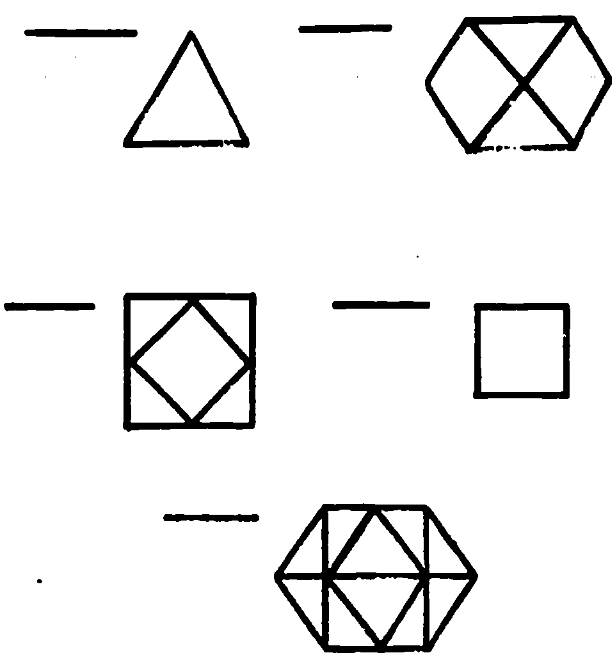


Space Figure

3.

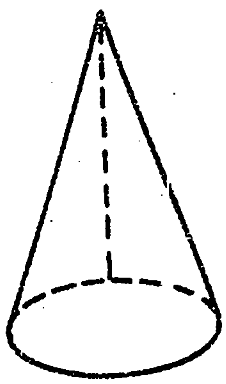


Possible Shadows

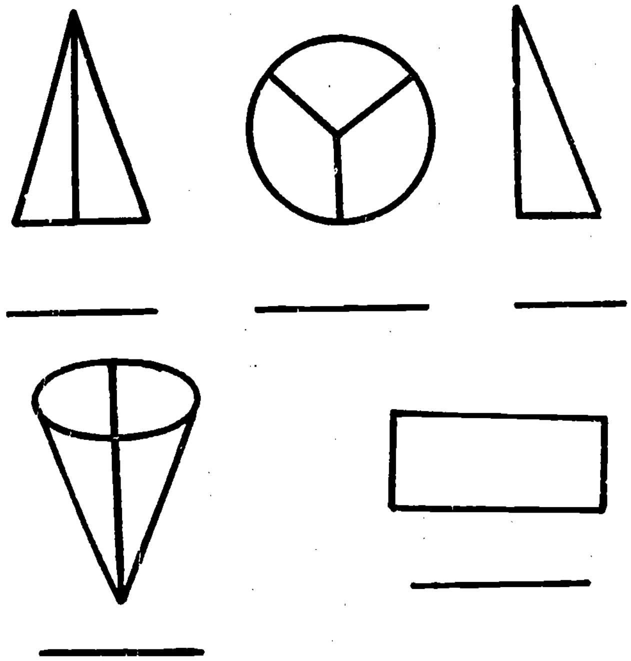


Space Figure

4.

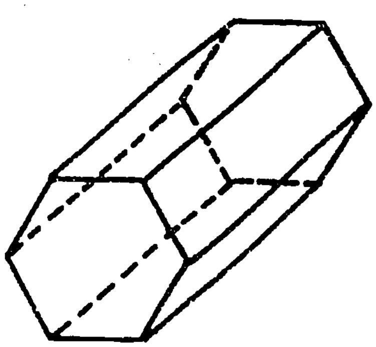


Possible Shadows

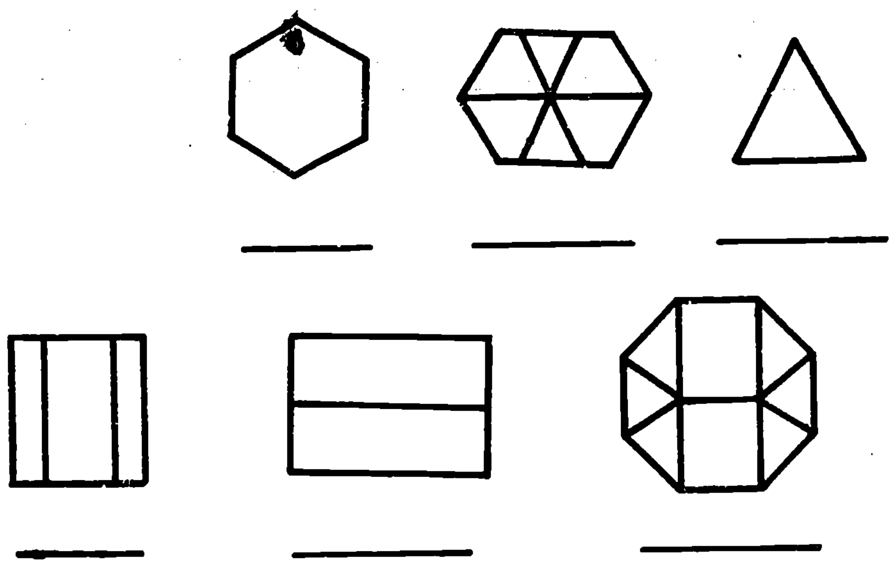


Space Figure

5.

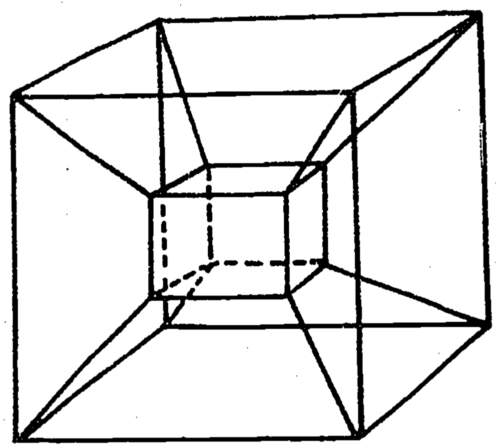


Possible Shadows

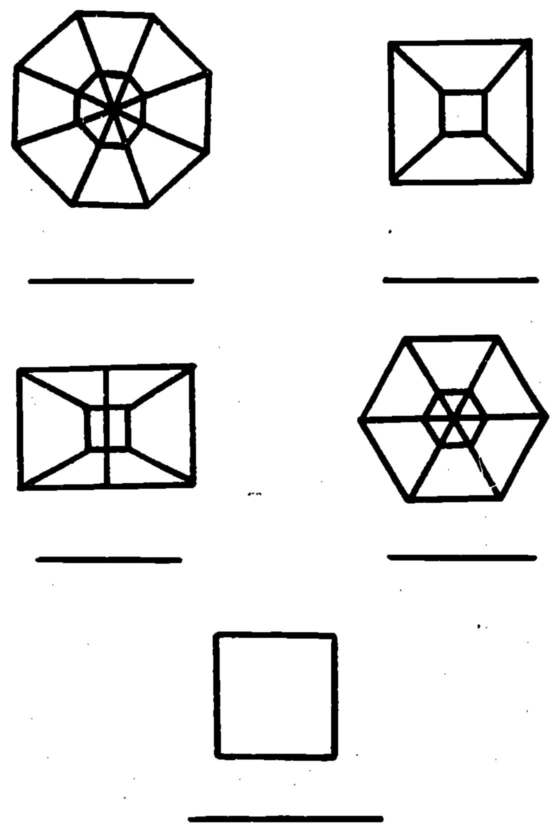


Space Figure

6.



Possible Shadows

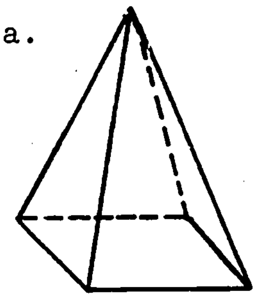
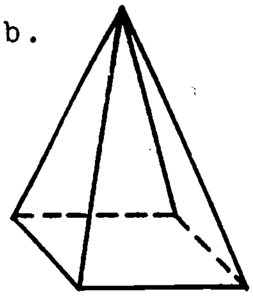
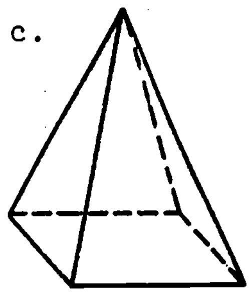
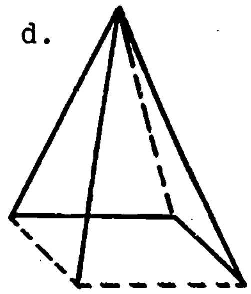


This brings to an end our first excursion into the world of geometry. Our next journey will take us into the world of two dimensions where we will investigate some of the properties of the shadow shapes you have seen in this last lesson. We hope you have enjoyed your excursion. Glad to have you aboard!

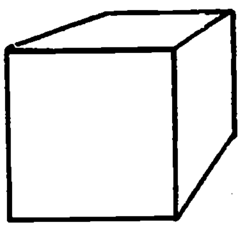
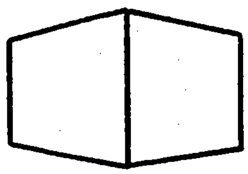
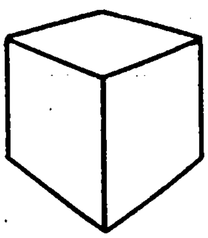
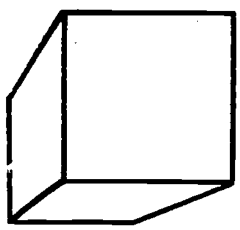
Before you unfasten your seatbelts

POINT

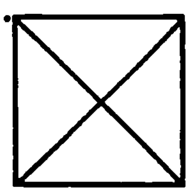
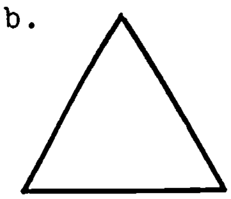
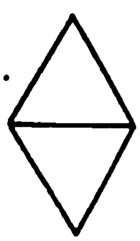
1. Which of the following sketches of a square pyramid have all the hidden lines drawn correctly? ________________

a. 
b. 
c. 
d. 

2. Which of the following cubes were drawn using the one-point perspective technique? ____________

a. 
b. 
c. 
d. 

3. Which of the following shadow shapes could be produced by shining a light on a straw model of a <u>square</u> pyramid held in different positions? ____________

a. 
b. 
c. 
d.